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5/15 (水) Proofs.

Lem 5.12 Let $G =$ cfg, $M =$ psp ($L(M) \subseteq L(G)$). If $\$\gamma|xy \overset{\theta}{\Rightarrow} \$\delta|y\$$ in M, then $\gamma^R \overset{\tau(\theta)}{\underset{lm}{\Rightarrow}} \delta^R x$ in G, $|\theta| = |\tau(\theta)| + |x|$.

Proof Induction on $|\theta| \in \mathbb{N}$.

i) Basis $|\theta| = 0$ $\theta = \varepsilon$. $\$\gamma|xy\$ \overset{\varepsilon}{\Rightarrow} \$\delta|y\$$, $\gamma = \delta$, $xy = y$, $x = \varepsilon$, $\tau(\theta) = \varepsilon$. If $\gamma|y = \$\gamma|y$ in G

Then $\gamma^R \overset{\varepsilon}{\Rightarrow} \gamma^R$ in G. $|\theta| = 0 = |\tau(\theta)| + |x| = 0 + 0 = 0$.

ii) Induction assume $\theta = r\theta'$ where $r \in \Gamma$, $\theta' \in \Gamma^*$ $\therefore \theta \in \Gamma^+$

Assume L5.12 is true (valid) for $\theta' \in \Gamma^*$. We must prove the L5.12 is true for $\theta \in \Gamma^+$ ($\theta = \gamma \cdot \theta'$)

ii.1) $r = A| \to \omega^R| \in \Gamma_{in} M$ $(A \to \omega \in P$ in $G)$

If $\$\gamma|xy\$ = \$\gamma' A|xy\$ \overset{A|\to\omega^R|}{\Rightarrow} \$\gamma'\omega^R|xy\$ \overset{\theta'}{\Rightarrow} \$\delta|y\$$ in M (If part of L5.12 가정)

$(\alpha\beta)^R = \beta^R\alpha^R$, $\alpha, \beta \in V^*$; $A^R = A$, $A \in V$

$(\gamma'\omega^R)^R \overset{\tau(\theta')}{\underset{lm}{\Rightarrow}} \delta^R x \wedge |\theta'| = \tau(\theta') + |x|$ by I.H.

$\gamma^R = (\gamma' A)^R = A\gamma'^R \overset{A\to\omega}{\Rightarrow} \omega\gamma'^R = (\gamma'\omega^R)^R$.

~~$\therefore \gamma^R = (\gamma' A)^R = A\gamma'^R \underset{lm}{\Rightarrow} \omega\gamma'^R = ($~~

$\gamma^R \overset{A\to\omega}{\underset{lm}{\Rightarrow}} (\gamma'\omega^R)^R \overset{\theta'}{\underset{lm}{\Rightarrow}} x\delta^R$ in G

$|\theta'| = |\tau(\theta')| + |x|$ $|\theta| = |\theta'| + |A \to \omega| = |\tau(\theta')| + 1 + |x| = \tau(\theta) + |x|$

QED.

ii.2) $r = a|a$