1) 3/21 (화) Monoid, 1.6 Rewriting Systems

$(V^*, \cdot, \varepsilon)$ is a free monoid. $(M = \underline{\underline{V}}^*)$

V: a set of vocabulary. $a \in V$ voc. (atom)

concatenation

$\cdot : V^* \times V^* \to V^*$ $\cdot$ is a binary operation on V^* (closed) — ① algebraic system

ε

$\forall x, y, z \in V^*$: $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ associative ② semigroups

$\forall x \in V^*$ $x \cdot \varepsilon = \varepsilon \cdot x = x$ ε: identity ③ monoid

finiteness =
X: $\underline{n} \in \mathbb{N}$
$x \in V^*$
$x = (a_1, a_2, \cdots, a_n)$
$= a_1 a_2 \cdots a_n$
justaxaposed

ex) $(2^{A \times A}, \cdot, id_A)$ is a monoid.

$(2^{V^*}, \cdot, \{\varepsilon\})$ is an (extended) (induced) free monoid - Fact 1.3

$\cdot : 2^{V^*} \times 2^{V^*} \to 2^{V^*}$

$V^{\{1,2,\cdots,n\}} \triangleq \{1,2,\cdots,n\} \to V \triangleq V^n$ $\div$ $\bigcup_{n \in \mathbb{N}} V^{\{1,2,\cdots,n\}}$ (V^n ‖) $\triangleq Strings(V)$ $= V^*$

2^* vs $2^{\mathbb{N}} (= \cancel{2^{\infty}})$

$|2^*| = |\mathbb{N}|$ $<$ $|2^{\mathbb{N}}|$

countably infinite — uncountable

$2^* = \cancel{\bigcup_{k=0}^{\infty}} \underline{\{1, \cdots, k\}} \to \{0, 1\}$ $= \bigcup_{n \in \mathbb{N}} 2^n$ (2)

하도록 $2^{\mathbb{N}}$ $\{1, 2, \cdots\} \to \{0, 1\}$

$= \phi \to \{0,1\} \cup \{1\} \to \{0,1\} \cup \{1,2\} \to \{0,1\} \cup \{1,2,3\} \to \{0,1\}$

$\cup \cdots \cup \{1,2,\cdots,n\} \to 0,1 \cup \{1,2,\cdots,n,n+1\} \to \{0,1\} \cup \cdots$

$= \{\varepsilon\} \cup \{0,1\} \cup \{00, 01, 10, 11\} \cup \{000, 001, \cdots, 111\} \cup$

$\cdots \{\underbrace{0\cdots0}_{n}, 0\cdots01, \cdots\}$ size: 2^n

$O(\underline{n}^k)$ vs $O(\underline{k}^n)$ n: finite

polynomial tractable — exponential intractable — $\cancel{ok}$ ok

$\left[\begin{array}{l} n \to \infty, \bigcup n^k - \text{countable} \\ n \to \infty, k^{\infty} - \text{uncountable} \end{array} \right.$

③ $x \in V^*$ (vs V^+)

	ele	set
single	$a \in V$ symbol	V vocabulary
a seq. of symbol	$x \in V^*$ string	V^*, $L \subseteq V^*$ language

skip

1.3 RAM & 1.4. Decision Problem & 1.5 Computational complexity

Let V be a vocabulary

① V : vocabulary (alphabet) atom

② $a \in V$: symbol (~~string~~) (문자)

③ V^* : an (infinite) sum of (all) strings

$x \in V^*$: a string over V = a (finite) sequence of symbols (문자열)

universe of string

④ $L \subseteq V^*$: a language (언어) (= a set of strings = a seq. of symbols

문자열 : $\mathbb{N} \to V$

수열 : $\mathbb{N} \to \mathbb{R}$

1.6 Rewriting Systems — grammar

(V, P) finite vs $G = (N, T, P, S)$

$N \cup T = V$, $S \in N (\subseteq V)$, $P \subseteq A \times V^*$

$N \cap T = \emptyset$ If $(A, \alpha) \in P$, $A \to \alpha \in P$.

V : an alphabet

$P \subseteq V^* \times V^*$

If $(w_1, w_2) \in P$, $(w_1, w_2 \in V^*)$ we write $w_1 \to w_2$ (or $w_1 \to w_2 \in P$)

rewrite strings in V^* $\quad$ rewrite $\subset V^* \times V^*$ $\in$

$\Rightarrow \subseteq V^* \times V^*$

$G = (V, P)$

$\gamma = \alpha w_1 \beta$, $w_1 \to w_2 \in P$.

$\gamma (= \alpha w_1 \beta)$ rewrites $\alpha w_2 \beta (= \delta)$, written

$\gamma \Rightarrow \delta$ (or $\alpha w_1 \beta \underset{G}{\overset{w_1 \to w_2}{\Rightarrow}} \alpha w_2 \beta$)

rewrite

compare P vs $\Rightarrow$ $\quad P, \Rightarrow \subseteq V^* \times V^*$ finite rel.

P : finite $\quad \Rightarrow$: infinite (induced relation from by P)

Recursive definition of $\underset{G}{\overset{\pi}{\Rightarrow}}$ for $\pi \in P^*$

$\underset{G}{\overset{\varepsilon}{\Rightarrow}}$ $\quad \varepsilon$

$\underset{G}{\overset{\pi}{\Rightarrow}} \doteq \underset{G}{\overset{\gamma}{\Rightarrow}} \overset{\pi'}{\Rightarrow}$ where $\gamma \in P$, $\pi' \in P^*$, $\gamma \pi' \in P^+$

Re def $\mathbb{N}$

1. $0 \in \mathbb{N}$

2. $n \in \mathbb{N} \Rightarrow n^{+1} \in \mathbb{N}$