3/16 (木) Cantor's Diagonal Argument.

1.2 ~~Rewriting systems~~ Languages

Review

Relation $R \subseteq A \times B$

function $f: A \to B$

operation $\oplus: A \times A \to A$

※ $A^B = \{f \mid f: B \to A\}$

$V^{\{1,2,\dots,n\}} \triangleq V^n$

길이 n짜리 V의 string

2^n .. 길이 n짜리 ~~binary~~ 이진수

Cantor's Diagonal Argument. (1891; 1874)

Russel's Paradox (1901) $R = \{x \mid x \notin x\}$

Gödel's Incompleteness Theorem (1931)

Halting Problem

$\{0,1\}^{\mathbb{N}} = \{f: \mathbb{N} \to \{0,1\}\} \cong 2^{\mathbb{N}}$ ... 무한이진수 (자연수의 부분집합)

$s \notin \{s_1, s_2, \dots\}$ but $s \in 2^{\mathbb{N}}$ $\therefore 2^{\mathbb{N}} \neq \{s_1, s_2, \dots\}$

complement of diagonal elements $\therefore |2^{\mathbb{N}}| > |\mathbb{N}|$

1.2 Language

① $(M, \cdot)$ is an algebraic system, if M is a set.

$\cdot : M \times M \to M$

② $(M, \cdot)$ is a semigroup, if $\cdot$ is associative.

$\forall a,b,c \in M: a \cdot (b \cdot c) = (a \cdot b) \cdot c$

binary-op $\Downarrow$ n-ary op

Ex) $(\mathbb{N}, +)$ is a semigroup. $1 + (2+3) = (1+2)+3$

Pascal $(((1+2)+3)+\dots+100)$

$= 101 \times 50 = 5050$

$= \sum_{i=1}^{100} i$ - n-ary prefix operator

※ $\mathbb{N}_0 = \{0, 1, \dots\}$

$\mathbb{N}_1 = \{1, 2, \dots\}$

$\mathbb{N}_0 \neq \mathbb{N}_1$, $\mathbb{N}_0 \supsetneq \mathbb{N}_1$

$\mathbb{N}_0 \triangleq \mathbb{N}_1$

$(M, \cdot, e)$ is a monoid, if $(M, \cdot)$ is a semigroup

e: is an identity element

$\forall a \in M \quad a \cdot e = e \cdot a = a$

Ex) $(\mathbb{N}, +, 0)$

V^* vs $V^{\mathbb{N}}$ → (union) infinite string

↑
union of finite string (infinite) over V

$= \bigcup_{i \in \mathbb{N}_0} V^i$

$|\mathbb{N}| = |\mathbb{N}|$ $|2\mathbb{N}| = |\mathbb{N}|$

$= 2 \cdot |\mathbb{N}| =$

증명 HW #1

$V^* \longleftrightarrow \mathbb{N}$

length first, lex order next

$\varepsilon, (a, \cdots, z)$ $(aa, ab, \cdots zz$

$(0,0)$ $(1,0)$ $(2,0)$
$(0,1)$ $(1,1)$ $(2,1)$
$(0,2)$ $(1,2)$ $(2,2)$

$\underline{|\mathbb{N}^2| = |\mathbb{N}| \times |\mathbb{N}| = |\mathbb{N}|}$ 증명. f:

$|\mathbb{N}^k| = |\mathbb{N}|$ $\big((0,0)\big)\big((0,1),(1,0)\big)\big((0,2),(1,1),(2,0)\big)$
$\big((0,3),(1,2),\cdots(3,0)\big)\cdots$

sum first, lexicographic order next

Quiz #2 $\mathbb{N} \times \mathbb{N} \xrightarrow{f} \mathbb{N}$ 와
$\mathbb{N} \times \mathbb{N} \xleftarrow{f^{-1}} \mathbb{N}$ 를 정의하시오

영어의 universe $\{a, b, \cdots, z\}^*$

한국어 〃 $\subset$ $\{ㄱ, \cdots ㅎ, ㅏ, \cdots ㅣ\}^*$

(초·중·종)*

$19 \times 21 \times 28 = 1,1xxx$ 자 — unicode