

# 3/16 (木) Cantor's Diagonal Argument.

Review

Relation  $R \subseteq A \times B$   
 function  $f: A \rightarrow B$   
 operation  $\oplus: A \times A \rightarrow A$

1. ~~Repeating systems~~  
 2. Languages

$\times A^B = \{f \mid f: B \rightarrow A\}$   
 $\forall \{1, 2, \dots, n\} \triangleq \sqrt{n}$   
 $\forall$  길이 n짜리  $\forall$ 의 string  
 $\forall 2^n \dots$  길이 n짜리 ~~string~~  
 이진수

Cantor's Diagonal Argument. (1891; 1894)

Russel's Paradox (1901)  $R = \{x \mid x \notin x\}$

Gödel's Incompleteness Theorem. (1931)

Halting Problem

$\{0, 1\}^{\mathbb{N}} = \{f: \mathbb{N} \rightarrow \{0, 1\}\} \cong 2^{\mathbb{N}} \dots$  무한이진수 (자연수의 부분집합)

$s \in \{s_1, s_2, \dots\}$  but  $s \in 2^{\mathbb{N}} \therefore 2^{\mathbb{N}} \neq \{s_1, s_2, \dots\}$   
 complement of diagonal elements  $\therefore |2^{\mathbb{N}}| > |\mathbb{N}|$

## 1.2 Languages

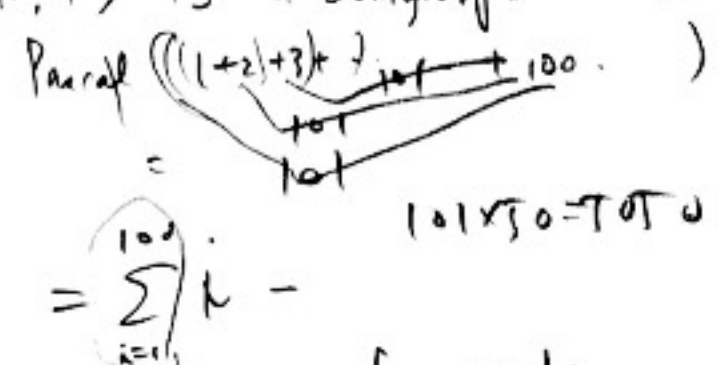
$(M, \cdot)$  is an algebraic system, if  $M$  is a set.

$\therefore M \times M \rightarrow M$

$(M, \cdot)$  is a semigroup, if  $\cdot$  is associative.

$\forall a, b, c \in M: a \cdot (b \cdot c) = (a \cdot b) \cdot c$

Ex)  $(\mathbb{N}, +)$  is a semigroup.  $1 + (2 + 3) = (1 + 2) + 3$



binary-op  
 $\Downarrow$   
 n-ary op

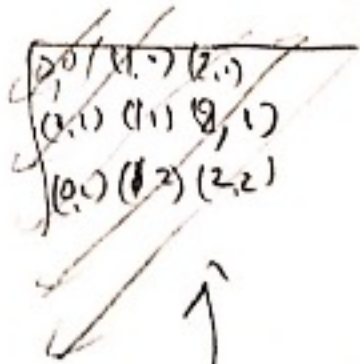
$\times N_0 = \{0, 1, \dots\}$   
 $N_1 = \{1, e, \dots\}$   
 $N_0 \neq N_1, N_0 \not\supseteq N_1$   
 $N_0 \cong N_1$

$(M, \cdot, e)$  is a monoid, if  $(M, \cdot)$  is a semigroup  
 $e$ : is an identity element

$\forall a \in M \quad a \cdot e = e \cdot a = a$

Ex)  $(\mathbb{N}, +, 0)$

$V^*$  vs  $V^{\mathbb{N}}$   
 ↑  
 Union of finite string (infinite) over  $V$  → (union) infinite string



$\bigcup_{i \in \mathbb{N}_0} V^i$   
 $|N| = |N| \quad |2N| = |N|$   
 $\uparrow$   
 $= 2 \cdot |N| =$

$|N^2| = |N| \times |N| = |N|^2$   
 $|N^k| = |N|^k$   
 (0,0), (0,1), (1,0), (0,2), (1,1), (2,0), (1,2), (2,1), (3,0), ...

↑  
 (HW #1)  
 $V^* \leftrightarrow \mathbb{N}$   
 length first, lex order next

#2  
 Quiz  $\mathbb{N} \times \mathbb{N} \leftrightarrow \mathbb{N}$   
 $\mathbb{N} \times \mathbb{N} \leftarrow \mathbb{N}$   
 sum first, lexicographic order next

$\Sigma(a_1, \dots, a_n)$   $(a_1, a_2, \dots, a_n)$

