3/4 (火) Free Monoid over V $(V^*, \circ, \varepsilon)$

Relation $R \subseteq A \times B$ or $R \subseteq A \times A$

1. equivalent ref. symm. trans.
2. partial order ( ) asymm. trans.

代数系 algebraic system.

$(\mathbb{N}, =)$

$(\mathbb{N}, <), (\mathbb{N}, \leq)$ — total order

$(2^S, \subseteq)$ — partial

Function $f: A \to B$

1. $f \subseteq A \times B$
2. uniqueness $a \in A$, $\exists 1$ $f(a) \in B$ ($f(a) = b$)
3. total $\forall a \in A$, $\exists$ $f(a) \in B$

$\forall a \in A, \exists_1 b = f(a) \in B$

closed

$(\mathbb{N}, +)$ $+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$(\mathbb{N}, \times)$

$(\mathbb{N}, -)$ $- : \mathbb{N} \times \cancel{\mathbb{N}} \to \mathbb{N}$ ~~closed~~

$(\cancel{\mathbb{N}, \div})$

$(\mathbb{Z}, -)$ $- : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$

$(\cancel{\mathbb{Z}, \div})$

유리수 $Q - \{0\}$

$(\mathbb{Q}, /)$ $/ : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$

Cond of 유클리드

피타고라스

$x^2 + y^2 = z^2$ — $\sqrt{2}$ π

무리수

※ operation (연산): $f: A \to A$ unary op.

$: A \times A \to A$ binary op

$: \underbrace{A \times A \times \cdots \times A}_{n} \to A$ n-ary op.

$f: A \to B$

① injection $f(a) = f(b) \Rightarrow a = b$

(1:1) or $a \neq b \Rightarrow f(a) \neq f(b)$ (대우명제)

단사 $|A| \leq |B| \Leftrightarrow |A| \cancel{>} |B|$

$f^{-1}: B \to A$ unique

② surjection (onto) 전사 $f(A) = B$ $(f(A) \subseteq B)$

$|A| \geq |B| \Leftrightarrow |A| \not< |B|$

$f^{-1}: B \to A$ total

함수의 image가 꽉 차다.

③ bijection $|A| = |B|$

$f^{-1}: B \to f$ is a function

$f: A \leftrightarrow B$ the set B is defined w.r.t. the set A and f

" A " " " B " f^{-1}

alg. system ?

$(\mathcal{U}, \cancel{\text{bijection}})$

$A \cong_f B$

set isomorphic w.r.t. f

$(\forall a \in A, \exists_1 f(a) = b \in B)$

$\wedge (\forall b \in B, \exists_1 f^{-1}(b) = a \in A)$

set isomorphism

$(\mathbb{U}, \cong)$ $\quad \cong \subseteq \mathbb{U} \times \mathbb{U}$

$\forall A, B \in \mathbb{U}: A \cong B \iff \exists f \wedge A \xleftrightarrow{f} B$ $\quad \exists f \wedge f$ is bijective

$\exists f: A \to B$

명제 P ⇒ 원래 가정의 P
조건 (cond) 약 weak
강 strong

Fact

$\cong$ is equivalent rel.

① $\forall A \in \mathbb{U}$: $A \cong A$

② $\forall A, B \in \mathbb{U}$: $A \cong B \Rightarrow B \cong A$

$R \Rightarrow$ 옴

③ $\forall A, B, C \in \mathbb{U}$: $A \cong B,\ B \cong C$

$pow(\mathbb{U}) = \{\{\{\}\}, \{\{0\}, \{100\}, \dots\}, \{\{0,1\}, \{1,2\}, \dots\}, \dots\}$ (0, 1, 2)

{개똥}
{바나나}

$R \subseteq A \times A$ is equi. $[a]_R = \{b \in A \mid aRb\}$ equi. class

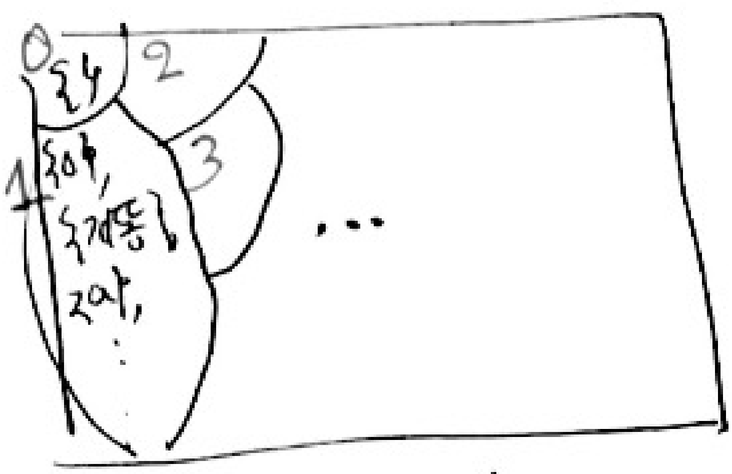
$[\{\}]_{\cong}$ ≅ $[\{개똥\}]_{\cong}$

$\cong \quad \mathbb{N}_0 = \{0, 1, 2, \dots$

$= [\{0\}]_{\cong} = [\{1\}_{\cong}] = \dots$

$= 1$

$[A]_{\cong} = |A|$



string over {a, b} of length n.

$A^B = \{f \mid f: B \to A\}$ vs. $B^A = \{f \mid f: A \to B\}$ $\quad$ {aa..a, aba..a, ⋯, bb..b}

⇕

Ex) $f: \{1, 2, \dots, n\} \to \{a, b\} \leftrightarrow \{0, 1\}$

binary string of length n

$\{a, b\}^{\{1, 2, \dots, n\}}$

$= \{\{f(1)=a, f(2)=a, \dots, f(n)=a\},$ $\quad$ $\{(a, a, \dots, a)$

$\{f(1)=b, f(2)=a, f(3)=a, \dots f(n)=a\}$ $\quad \Leftrightarrow (b, a, \dots, a)$

$\{f(1)=a, f(2)=b, f(3)=a, \dots f(n)=a\},$ $\quad (a, b, a, \dots, a)$

$\dots,$ $\quad \dots$

$f(1)=b, f(2)=b, \dots, f(n)=b\}$ $\quad (b, b, b, \dots,$

$\{0,1\}^{\{1,2,\dots,n\}} \triangleq \{0,1\}^n \qquad n \in \mathbb{N}_0$

$\{0,1\}^0 = \{\varepsilon\}$ — 1개 $= 2^0$개

$\{0,1\}^1 = \{0,1\}$ — 2개 $= 2^1$개

$\{0,1\}^2 = \{00, 01, 10, 11\}$ — 4개 $= 2^2$개

$\{0,1\}^3 = \{000, 001, \dots, 111\}$ — 8개 $= 2^3$개

$\vdots$

$\{0,1\}^n = \{ \dots \}$ — 2^n개

$\vdots$

$\cup$

$\{0,1\}^* = \bigcup_{i=0}^{\infty} \{0,1\}^i$ ... countably infinite

$(\{0,1\}^*, \cdot)$ is an algebraic system

$\forall x, y \in \{0,1\}^*,\ x \cdot y = xy$ — juxtaposed

Ex $001 \cdot 101111 = 001101111$

i) closed — a.s. $(V^*, \cdot)$

ii) associative ... semigroup $(V^*, \cdot)$

iii) identity ... free monoid $(V^*, \cdot, \varepsilon)$