3/9(木) Equivalence relation, partial order, function, cardinality of sets

Relation $R \subseteq A \times A$ is equivalence, if
R is (1) reflexive, (2) symmetric, and (3) transitive.

Def equivalence class $a \in A$.

$\underline{[a]_R = \{b \mid aRb\}}$ Fact 1 $a \in [a]_R$

Fact 2 $\bigcup_{a \in A} [a]_R (= [a_1]_R \cup [a_2]_R \cup \cdots \cup [a_8]_R) = A$

exhaustive

Ex) $A = \{a_1, a_2, \ldots, a_8\}$

Fact 3 If aRb, then $[a]_R = [b]_R$ ...

Fact 4 If $a \not R b$, then $[a]_R \cap [b]_R = \emptyset$

한 identifier를 갖는 P.L.

예: 사랑 $\subseteq$ 사람 $\times$ 사람 사랑이 equivalent 라면

문학 작품: 로미오와 줄리엣. $[\text{로미오}]_{\text{사랑}}$ = (동등) ↓
$\|$ {로 ... } 분당이 생긴다
$[\times\times]_{\text{사랑}}$ partition

Partition of a set A. ① exhaustive ② mutually disjoint!

$Par(A) = \{A_1, A_2, \ldots, A_k\}_R$

$1 \le {}^\forall i \le k$: $A_i \subseteq A$, $\bigcup_{i=1} A_i = A$.

$1 \le i \ne j \le k$ $A_i \cap A_j = \emptyset$

Let $|A| = n$. $R \approx O(n^2)$ $Par(A) \approx O(n)$
$|Par(A)| \le n$

Ex) $A = \{a_1, a_2, \ldots, a_8\}$

$Par(A) = \{[a_1]_R, [a_2]_R, \ldots, [a_8]_R\}$

혹시 $[a_1]_R \overset{?}{=} [a_2]_R$,
$\cap \overset{?}{=} \emptyset$

$Par(A) = \{\{a_1, a_2, a_3\}, \{a_4, a_5, a_6, a_7, a_8\}\}$

Ex) $= \subseteq \mathbb{N} \times \mathbb{N}$. $[n]_=$ ($n \in \mathbb{N}$ 일때) $\mathbb{N} = \{0, 1, 2, \ldots\}$

$[0]_= = \{0\}$, — 자연수

2) Partial order

1) (ref.) 2) anty symmetric 3) transitive

Ex) $<\ \subseteq \mathbb{N}\times\mathbb{N}$ antisym. $2<3 \wedge 3<2 \Rightarrow 3=2$ irreflexive p.o.

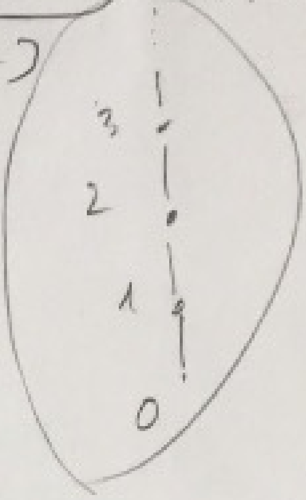
$2<3 \mid \underline{3<2}$

tran $2<3 \wedge 3<5 \Rightarrow 2<5$

$\leq\ \subseteq \mathbb{N}\times\mathbb{N}$

$3\leq 3 \wedge 3\leq 3 \Rightarrow 3=3$ ref. p.o

$\cancel{2\leq 3 \wedge 3\leq 2 \Rightarrow 2=3}$



p.o $<, \leq$, linear partial order (total) order

Let $\mathbb{U}$ be a set of (all) sets.

$2^{\mathbb{U}} \triangleq \{A \mid A \subseteq \mathbb{U}\}$

$= \{\cancel{A \in 2^{\mathbb{U}}} \mid A \subseteq \mathbb{U}\}$

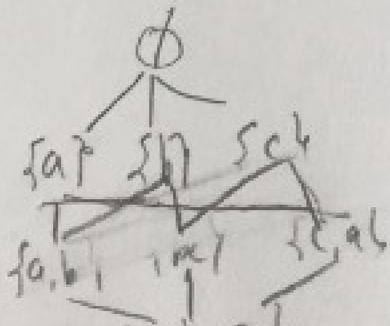
Let $A, B \subseteq \mathbb{U}$. Then we define $A\,R\,B$ if $|A| = |B|$.

Let $A, B \subseteq \mathbb{U}$ Then " $A \subseteq B$ if $\forall a \in A,\ a \in B$.

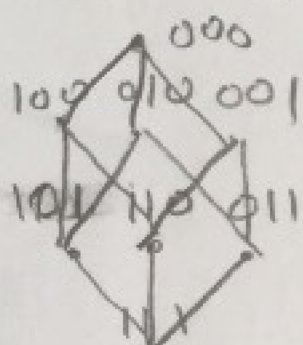
$A, B \in 2^{\mathbb{U}}$. $\subseteq$ .. paritial order

Ex $\mathbb{U} = \{a, b, c\}$

$2^{\mathbb{U}} = P(\mathbb{U}) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$



Powerset lattice of degree 3



binary number of length 3

a function from the set A to the set B.

f

$f: A \to B \xleftrightarrow{\cancel{\ }} R \subseteq A \times B$

1. relation

2. unique $\forall a \in A$, $\exists!$ $\underline{f(a)} \in B$ $\qquad R(a) \cancel{\in} B$ $\subseteq B$ (But multiple value returning function in C or Java)

3. total $\forall a \in A$ $\exists f(a) \in B$

$\forall a \in A$ $\exists!$ $f(a) \in B$.

$\parallel$ b instead of $\{b\}$ $\qquad R(a) = \{b_1, b_2, \ldots, b_n\} \subseteq B$

$A = B$ vs $A \xleftrightarrow[f]{} B$