Left and Right Parsers

Consider a leftmost derivation

$$S \Rightarrow^*_{lm} xY\gamma \Rightarrow_{lm} x\beta\gamma \Rightarrow^*_{lm} xy\gamma \Rightarrow^*_{lm} xyz,\ Y \in N \cup \Sigma,\ Y \Rightarrow^* y \in \Sigma^*.$$

Two cases in $Y \Rightarrow^* y \in \Sigma^*$.

i) $Y \in N$, $Y \rightarrow \beta \in P$, and $\beta \Rightarrow^* y \in \Sigma^*$. ($Y \Rightarrow^+ y$)

$$S \Rightarrow^*_{lm} xY\gamma \Rightarrow_{lm} x\beta\gamma \Rightarrow^*_{lm} xy\gamma \Rightarrow^*_{lm} xyz.$$

$$\$S \mid xyz\$ \Rightarrow^* \$\gamma^R Y \mid yz\$ \Rightarrow^{Y\rightarrow\beta} \$\gamma^R\beta^R \mid yz\$ \Rightarrow^* \$\gamma^R \mid z\$ \Rightarrow^* \$ \mid \$.$$

ii) $Y \in \Sigma$, $Y = y$. ($Y \Rightarrow^0 y$ or $Y = y$)

$$S \Rightarrow^*_{lm} xY\gamma = xy\gamma \Rightarrow^*_{lm} xyz.$$

$$\$S \mid xyz\$ \Rightarrow^* \$\gamma^R Y \mid yz\$ = \$\gamma^R y \mid yz\$ \Rightarrow^{y=y} \$\gamma^R \mid z\$ \Rightarrow^* \$ \mid \$.$$

i) $Y \in N, Y \rightarrow \beta \in P$, and $\beta \Rightarrow^* y$.

$$S \Rightarrow^*_{lm} xY\gamma \Rightarrow_{lm} x\beta\gamma \Rightarrow^*_{lm} xy\gamma \Rightarrow^*_{lm} xyz.$$

$$\bullet S \Rightarrow^*_{lm} x\bullet Y\gamma \Rightarrow_{lm} x\bullet\beta\gamma \Rightarrow^*_{lm} xy\bullet\gamma \Rightarrow^*_{lm} xyz\bullet.$$

$$(S, xyz) \Rightarrow^* (Y\gamma, yz) \Rightarrow^{Y\rightarrow\beta} (\beta\gamma, yz) \Rightarrow^* (\gamma, z) \Rightarrow^* (\varepsilon, \varepsilon).$$

$$[S, xyz] \Rightarrow^* [\gamma^R Y, yz] \Rightarrow^{Y\rightarrow\beta} [\gamma^R\beta^R, yz] \Rightarrow^* [\gamma^R, z] \Rightarrow^* [\varepsilon, \varepsilon].$$

$$\$S \mid xyz\$ \Rightarrow^* \$\gamma^R Y \mid yz\$ \Rightarrow^{Y\rightarrow\beta} \$\gamma^R\beta^R \mid yz\$ \Rightarrow^* \$\gamma^R \mid z\$ \Rightarrow^* \$ \mid \$.$$

ii) $Y \in \Sigma, Y = y$.

$$S \Rightarrow^*_{lm} xY\gamma =^{Y=y} xy\gamma \Rightarrow^*_{lm} xyz.$$

$$\bullet S \Rightarrow^*_{lm} x\bullet Y\gamma =^{Y=y} x\bullet yz \Rightarrow^{y=y} xy\bullet\gamma \Rightarrow^*_{lm} xyz\bullet.$$

$$(S, xyz) \Rightarrow^* (Y\gamma, yz) =^{Y=y} (y\gamma, yz) \Rightarrow^{y=y} (\gamma, z) \Rightarrow^* (\varepsilon, \varepsilon).$$

$$[S, xyz] \Rightarrow^* [\gamma^R Y, yz] =^{Y=y} [\gamma^R y, yz] \Rightarrow^{y=y} [\gamma^R, z] \Rightarrow^* [\varepsilon, \varepsilon].$$

$$\$S \mid xyz\$ \Rightarrow^* \$\gamma^R Y \mid yz\$ =^{Y=y} \$\gamma^R y \mid yz\$ \Rightarrow^{y=y} \$\gamma^R \mid z\$ \Rightarrow^* \$ \mid \$.$$

Consider a rightmost derivation in normal and reversed order.

$$S \Rightarrow^*_{rm} \alpha Y z \Rightarrow_{rm} \alpha\beta z \Rightarrow^*_{rm} \alpha y z \Rightarrow^*_{rm} xyz.$$

$$xyz \Leftarrow^*_{rm} \alpha y z \Leftarrow^*_{rm} \alpha\beta z \Leftarrow_{rm} \alpha Y z \Leftarrow^*_{rm} S.$$

Two cases in $Y \Rightarrow^* y$.

i) $Y \in N$, $Y \rightarrow \beta \in P$, *and* $\beta \Rightarrow^* y.(Y \Rightarrow^+ y)$

$$xyz \Leftarrow^*_{rm} \alpha yz \Leftarrow^*_{rm} \alpha\beta z \Leftarrow_{rm} \alpha Y z \Leftarrow^*_{rm} S.$$

$$\$ \mid xyz\$ \Rightarrow^* \$\alpha \mid yz\$ \Rightarrow^* \$\alpha\beta \mid z\$ \Rightarrow^{\beta \leftarrow Y} \$\alpha Y \mid z\$ \Rightarrow^* \$S \mid \$.$$

ii) $Y \in \Sigma$, $Y = y$. $(Y \Rightarrow^0 y$ *or* $Y = y)$

$$xyz \Leftarrow^*_{rm} \alpha yz = \alpha Y z \Leftarrow^*_{rm} S.$$

$$\$ \mid xyz\$ \Rightarrow^* \$\alpha \mid yz\$ = \$\alpha \mid Yz\$ \Rightarrow^{Y=y} \$\alpha Y \mid z\$ \Rightarrow^* \$S \mid \$$$

i) $Y \in N$, $Y \to \beta \in P$, and $\beta \Rightarrow^* y$.

$$xyz \Leftarrow^*_{rm} \alpha yz \Leftarrow^*_{rm} \alpha\beta z \Leftarrow_{rm} \alpha Yz \Leftarrow^*_{rm} S.$$

$$\bullet xyz \Leftarrow^*_{rm} \alpha\bullet yz \Leftarrow^*_{rm} \alpha\beta\bullet z \Leftarrow_{rm} \alpha Y\bullet z \Leftarrow^*_{rm} S\bullet.$$

$$(\varepsilon, xyz) \Rightarrow^* (\alpha^R, yz) \Rightarrow^* (\beta^R\alpha^R, z) \Rightarrow^{\beta\leftarrow Y} (Y\alpha^R, z) \Rightarrow^* [S, \varepsilon].$$

$$[\varepsilon, xyz] \Rightarrow^* [\alpha, yz] \Rightarrow^* [\alpha\beta, z] \Rightarrow^{\beta\leftarrow Y} [\alpha Y, z] \Rightarrow^* [S, \varepsilon].$$

$$\$ \mid xyz\$ \Rightarrow^* \$\alpha \mid yz\$ \Rightarrow^* \$\alpha\beta \mid z\$ \Rightarrow^{\beta\leftarrow Y} \$\alpha Y \mid z\$ \Rightarrow^* \$S \mid \$.$$

ii) $Y \in \Sigma$, $Y = y$.

$$xyz \Leftarrow^*_{rm} \alpha yz =^{y=Y} \alpha Yz \Leftarrow^*_{rm} S.$$

$$\bullet xyz \Leftarrow^*_{rm} \alpha\bullet yz =^{y=Y} \alpha\bullet Yz \;{}^{Y=Y}\!\!\Leftarrow \alpha Y\bullet z \Leftarrow^*_{rm} S\bullet.$$

$$(\varepsilon, xyz) \Rightarrow^* (\alpha^R, yz) =^{y=Y} (\alpha^R, Yz) \Rightarrow^{Y=Y} (Y\alpha^R, z) \Rightarrow^* (S, \varepsilon).$$

$$[\varepsilon, xyz] \Rightarrow^* [\alpha, yz] =^{y=Y} [\alpha, Yz] \Rightarrow^{Y=Y} [\alpha Y, z] \Rightarrow^* [S, \varepsilon].$$

$$\$ \mid xyz\$ \Rightarrow^* \$\alpha \mid yz\$ =^{y=Y} \$\alpha \mid Yz\$ \Rightarrow^{Y=Y} \$\alpha Y \mid z\$ \Rightarrow^* \$S \mid \$$$

$\bullet S \Rightarrow^*_{lm} x\bullet A\gamma \Rightarrow_{\mathbf{lm}} x\bullet\beta\gamma \Rightarrow^*_{lm} xy\bullet\beta\gamma \Rightarrow^*_{lm} xy\bullet\gamma \Rightarrow^*_{lm} xyz\bullet.$

$\$S \mid xyz\$ \Rightarrow^* \$\gamma^R A \mid yz\$ \Rightarrow^{\text{produce } A \text{ as } \beta \text{ on } k:yz} \$\gamma^R\beta^R \mid yz\$ \Rightarrow^* \$\gamma^R \mid z\$ \Rightarrow^* \$ \mid \$.$

$\gamma^R A \mid \rightarrow_L \gamma^R\beta^R \mid$ *in left parser*

$[\gamma^R]_k[\gamma^R{\cdot}A]_k \mid k{:}yz \rightarrow_{LL(k)} [\gamma^R]_k[\gamma^R{\cdot}\beta{:}1]_k[\gamma^R{\cdot}(\beta{:}2)^R]_k\ldots[\gamma^R{\cdot}\beta^R]_k \mid k{:}yz.$

$\$S \mid xyz\$ \Rightarrow^* \$\gamma^R Y \mid yz\$ =^{Y=y=a} \$\gamma^R a \mid az\$ \Rightarrow^{\text{verify } a} \$\gamma^R \mid z\$ \Rightarrow^* \$ \mid \$.$

$\gamma^R a \mid a \rightarrow_L \gamma^R \mid$ *in left parser*

$[\gamma^R]_k[\gamma^R a]_k \mid a \rightarrow_{LL(k)} [\gamma^R]_k$

$S\bullet \Rightarrow^*_{rm} \delta A\bullet z \Rightarrow_{\mathbf{rm}} \delta\beta\bullet z \Rightarrow^*_{rm} \delta\bullet yz \Rightarrow^*_{rm} \bullet xyz.$

$\bullet xyz \Leftarrow^*_{rm} \delta\bullet yz \Leftarrow^*_{rm} \delta\beta\bullet z \Leftarrow_{\mathbf{rm}} \delta A\bullet z \Leftarrow^*_{rm} S\bullet.$

$\$ \mid xyz\$ \Rightarrow^* \$\delta \mid yz\$ \Rightarrow^* \$\delta\beta \mid z\$ \Rightarrow^{\text{reduce } \beta \text{ to } A \text{ on } k:z} \$\alpha A \mid z\$ \Rightarrow^* \$S \mid \$.$

$\delta\beta \mid \rightarrow_R A \mid$ *in right parser*

$[\delta]_k[\delta{\cdot}1{:}\beta]_k[\delta{\cdot}2{:}\beta]_k\ldots[\delta{\cdot}\beta]_k \mid k{:}z \rightarrow_{LR(k)} [\delta]_k[\delta]_k \mid k{:}z.$

$\$ \mid xyz\$ \Rightarrow^* \$\delta \mid yz\$ =^{y=a} \$\delta \mid az\$ \Rightarrow^{shift\ a} \$\alpha a \mid z\$ \Rightarrow^* \$S \mid \$$

$\delta \mid a \rightarrow_R \delta a \mid$ *in right parser*

$[\delta]_k \mid a \rightarrow_{LR(k)} [\delta]_k [\delta]_k [\delta \cdot a]_k \mid$