LL(k) Parser

Consider a leftmost derivation,

$$S \Rightarrow^*_{lm} uA\gamma \Rightarrow_{lm} u\alpha\beta\gamma \Rightarrow^*_{lm} uv\beta\gamma \Rightarrow^*_{lm} uvyz \text{ and } y \in First_k(\beta\gamma).$$

$$\bullet S \Rightarrow^*_{lm} u\bullet A\gamma \Rightarrow_{lm} u\bullet\alpha\beta\gamma \Rightarrow^*_{lm} uv\bullet\beta\gamma \Rightarrow^*_{lm} uvyz\bullet.$$

$$\$S \,|\, uvyz\$ \Rightarrow^*_L \$\gamma^R A \,|\, vyz\$ \Rightarrow_L^{A\to\alpha\beta} \$\gamma^R\beta^R\alpha^R \,|\, vyz\$ \Rightarrow^*_L \$\gamma^R\beta^R \,|\, yz\$ \Rightarrow^*_L \$ \,|\, \$$$

We ***define*** *$[A \to \alpha\bullet\beta, x]$ be a valid LL(k) item for a viable* ***suffix*** *$(\beta\delta)^R$.*
Let L_k denote a set of valid LL(k) items.

We define $Valid_k$: $(N\cup\Sigma)^* \to 2^{L_k}$ or $\langle\cdot\rangle_k$ for short.

$$Valid_k(\gamma) = \langle(\beta\delta)^R\rangle_k = \langle\delta^R\beta^R\rangle_k$$

$$= \{[A \to \alpha\bullet\beta, x] \in L_k \,|\, S \Rightarrow^*_{lm} uA\delta \Rightarrow_{lm} u\alpha\beta\delta,\ \gamma = (\beta\delta)^R,\ x \in First_k(\beta\delta)\}$$

Lemma *If* $[A \to \alpha\beta\bullet\gamma, x] \in \langle(\gamma\delta)^R\rangle_k = \langle\delta^R\gamma^R\rangle_k$, *then*

$[A \to \alpha\bullet\beta\gamma, y] \in \langle(\beta\gamma\delta)^R\rangle_k = \langle\delta^R\gamma^R\beta^R\rangle_k$

where $y \in First_k(\beta\gamma\delta) = First_k(\beta x)$

Proof $\exists S \Rightarrow^*_{lm} uA\delta \Rightarrow_{lm} u\alpha\beta\gamma\delta$, $x \in First_k(\gamma\delta)$

$\therefore [A \to \alpha\bullet\beta\gamma, y] \in \langle\delta^R\gamma^R\beta^R\rangle_k$ *where where* $y \in First_k(\beta x)$.

Cololary *If* $[A \to \alpha X\bullet\beta, x] \in \langle(\beta\delta)^R\rangle_k = \langle\delta^R\gamma^R\rangle_k$ *then*

$[A \to \alpha\bullet X\beta, y] \in \langle(X\beta\delta)^R\rangle_k = \langle\delta^R\gamma^R X\rangle_k$ *and* $y \in First_k(Xx)$.

Theorem *If* $[B \to \eta A\bullet\psi, x] \in \langle(\psi\delta)^R\rangle_k$ *and* $A \to \alpha \in P$, *then*

$[A \to \alpha\bullet, x] \in \langle(\psi\delta)^R\rangle_k$ *and vice versa.*

Proof $\exists S \Rightarrow^*_{lm} uB\delta \Rightarrow_{lm} u\eta A\psi\delta \Rightarrow^*_{lm} uvA\psi\delta \Rightarrow_{lm} uv\alpha\psi\delta$,

$x \in First_k(\psi\delta)$, $\eta \Rightarrow^* v$.

Let $K \subseteq L_k$.

Definition $\partial_{LL(k)}: 2^{L_k} \rightarrow 2^{L_k}$, ∂_k *or* ∂ *for short.*

(***desc***$_{LL(k)}$, ***desc***$_k$ *in text).*

$$\partial_k K = \{[B \rightarrow \eta\bullet, x] \in L_k \mid [A \rightarrow \alpha B\bullet\beta, x] \in K, B \rightarrow \eta \in P\}$$

Definition $\chi_{LL(k)}{}^X: 2^{L_k} \times (N \cup \Sigma) \rightarrow 2^{L_k}$, $\chi_k{}^X$ *or* χ^X *for short.*

(***passes-X***$_{LL(k)}$, ***passes-X*** *in text).*

$$\chi_k{}^X K = \{[A \rightarrow \alpha\bullet X\beta, y] \in L_k \mid [A \rightarrow \alpha X\bullet\beta, x] \in K, y \in First_k(X \cdot x)\}$$

If $K \in \langle\delta\rangle_k$, $\partial_k{}^* K \in \langle\delta\rangle_k$.

If $K \in \langle\delta\rangle_k$, $\chi_k{}^X K \in \langle X \cdot \delta\rangle_k$.

note that $\chi_k{}^X K \in \langle\delta \cdot X\rangle_k$ *in LR(k) parser.*

We define $\rho_{LL(k)} \subseteq (N\cup\Sigma)^* \times (N\cup\Sigma)^*$, ρ_k *or* ρ *for short.*

$\gamma\ \rho_k\ \delta$, *if* $\langle\gamma\rangle_k = \langle\delta\rangle_k$.

ρ_k *is an* ***equivalent*** *binary relation on* $(N\cup\Sigma)^*$.

$[\gamma]_k = \{\delta \in (N\cup\Sigma)^* \mid \gamma\ \rho_k\ \delta\}$ *equivalent class on* $(N\cup\Sigma)^*$.

We extend the domain of $\langle\cdot\rangle_k$ *from* $(N\cup\Sigma)^*$ *to* $2^{(N\cup\Sigma)^*}$.

We may write $\langle\delta\rangle_k$ *instead of* $\langle[\delta]_k\rangle_k$. *since* $\langle\delta\rangle_k = \langle[\delta]_k\rangle_k$.

We may write $\langle\delta\rangle_k$ *instead of* $[\delta]_k$, *or vice versa, since*

$\langle\delta\rangle_k \leftrightarrow^{1:1} [\delta]_k$.

An LL(k) ***state***	$\langle\delta\rangle_k \leftrightarrow^{1:1} [\delta]_k$.
A set of valid ***LL(k) items***	$\langle\delta\rangle_k$.
A set of valid ***viable suffices***	$[\delta]_k$.
A set of valid ***stack strings***	$[\delta]_k$.

Canonical Collection of <u>*LL(k) states*</u>*, C_k, and Q: $C_k \times (N \cup \Sigma) \to C_k$.*

(<u>sets of LL(k) items</u>, <u>equivalent classes of valid viable suffices</u>)

$\langle\varepsilon\rangle_k := \partial_k^{\,*}([S' \to S\bullet, \varepsilon]);\ C_k := \{\langle\varepsilon\rangle_k\};\ Q := \emptyset;$

repaeat

    for $\langle\delta\rangle_k \in C_k$ ***do***

        for $X \in N \cup \Sigma$ ***where*** $K^X = \{[A \to \alpha X \bullet \beta, x] \in \langle\delta\rangle_k\}$ ***do***

            $\langle\delta\cdot X\rangle_k := \partial_k^{\,*}(\chi_k^{\,X}(K^X));$

            $C_k := C_k \cup \langle\delta\cdot X\rangle_k;$

            $Q := Q \cup \{\langle\delta\cdot X\rangle_k \cdot X \to \langle\delta\rangle_k\}$

        od   ***od***

until *no more states are added to C_k.*

Fact & Construction of *verify actions of* Γ *in LL(k) parsing.*

If $[B \to \alpha \mathbf{X} \bullet \beta, x] \in \langle\gamma\rangle_k \in C_k$ *and* $\mathbf{X} \in N \cup \Sigma$, *then*

$^{\exists}[B \to \alpha \bullet \mathbf{X}\beta, y] \in \langle \mathbf{X}\cdot\gamma\rangle_k \in C_k$ *where* $y \in First_k(\mathbf{X}\cdot x)$,

Add $\langle\gamma\rangle_k\langle\gamma\cdot\mathbf{X}\rangle_k \mid \mathbf{X}\cdot z \to \langle\gamma\rangle_k \mid z \in \Gamma$, $z = k\text{-}1{:}y \in First_{k-1}(\mathbf{X}\cdot x)$.

Fact & Construction of *guess actions of* Γ *in LL(k) parsing.*

If $[B \to \alpha \mathbf{A} \bullet \beta, x] \in \langle\gamma\rangle_k \in C_k$ *and* $\mathbf{A} \to X_1\cdot X_2\cdot\ldots\cdot X_n \in P$, *then*

$^{\exists}[\mathbf{A} \to X_1\cdot\ldots\cdot X_{n-1}\cdot X_n \bullet, x] \in \langle\gamma\rangle_k$,

$[n \geq {}^{\forall}i \geq 1{:}\ {}^{\exists}[A \to X_1\cdot\ldots\cdot X_{i-1} \bullet X_i\cdot\ldots\cdot X_{n-1}\cdot X_n, y_i] \in \langle X_i\cdot\ldots\cdot X_{n-1}\cdot X_n\cdot\gamma\rangle_k$

where $y_i \in First_k(X_i\cdot x_i)$ *and* $x_n = x$,

$^{\exists}\langle\gamma\cdot X_i\cdot\ldots\cdot X_n\rangle_k\langle\gamma\cdot X_1\cdot\ldots\cdot X_{i-1}\cdot X_n\rangle_k \mid X_i\cdot z \to \langle\gamma\cdot X_1\cdot\ldots\cdot X_{i-1}\rangle_k \mid z \in \Gamma$

where $z \in First_k(X_{i+1}\cdot\ldots\cdot X_n\cdot y)]$.

Add $\langle\gamma\rangle_k\langle\gamma\cdot\mathbf{A}\rangle_k \mid \mathbf{y} \to \langle\gamma\rangle_k\langle\gamma\cdot X_n\rangle_k\langle\gamma\cdot X_n\cdot X_{n-1}\rangle_k\ldots\langle\gamma\cdot X_n\cdot\ldots\cdot X_1\rangle_k \mid \mathbf{y} \in \Gamma$.

LL(k) parser is a pdt $M_k = \{C_k, \Sigma, \Gamma, P, \tau, [\varepsilon]_k[S]_k, \{[\varepsilon]_k\}, \$, |\,)$ *where*

$\Gamma = \{[\gamma]_k[\gamma\cdot \mathbf{A}]_k \,|\, \mathbf{x} \to [\gamma]_k[\gamma\cdot X_n]_k[\gamma\cdot X_n\cdot X_{n-1}]\ldots[\gamma\cdot X_n\cdot X_{n-1}\cdot\ldots\cdot X_1]_k \,|\, \mathbf{x} \mid$

$[\mathbf{A} \to \bullet X_1\cdot X_2\cdot\ldots X_n, \mathbf{x}] \in \langle\gamma\cdot X_n\cdot X_{n-1}\cdot\ldots\cdot X_1\rangle_k\}$

$\cup \{[\gamma]_k[\gamma\cdot \mathbf{a}]_k \,|\, \mathbf{a}x \to [\gamma]_k \,|\, x \mid [A \to \alpha\bullet \mathbf{a}\beta, ax] \in \langle\gamma\cdot \mathbf{a}\rangle_k\}$

$\tau([\gamma]_k[\gamma\cdot \mathbf{A}]_k \,|\, \mathbf{x} \to [\gamma]_k[\gamma\cdot X_n]_k\ldots[\gamma\cdot X_n\cdot X_{n-1}\cdot\ldots\cdot X_1]_k \,|\, \mathbf{x})$

$= \mathbf{A} \to X_1\cdot\ldots\cdot X_n \in P,$

$\tau([\gamma]_k[\gamma\cdot \mathbf{a}]_k \,|\, \mathbf{a}x \to [\gamma]_k \,|\, x) \quad = \varepsilon.$

$[B \to \alpha \mathbf{A}\bullet\beta, y] \in \langle\gamma\rangle_k \qquad [B \to \alpha \mathbf{A}\bullet\beta, y] \in \langle\gamma\cdot A\rangle_k$

$[A \to X_1\cdot\ldots\cdot X_{n-1}\cdot X_n\bullet, y] \in \langle\gamma\rangle_k$

$[A \to X_1\cdot\ldots\cdot X_{n-1}\bullet\cdot X_n, y_n] \in \langle\gamma\cdot X_n\rangle_k \qquad [A \to \alpha \mathbf{a}\bullet\beta, x] \in \langle\gamma\rangle_k$

$[A \to X_1\cdot\ldots\cdot\bullet X_{n-1}\cdot X_n, y_{n-1}] \in \langle\gamma\cdot X_n\cdot X_{n-1}\rangle_k \qquad [A \to \alpha\bullet \mathbf{a}\beta, \mathbf{a}y] \in \langle\gamma\cdot \mathbf{a}\rangle_k$

…

$[A \to \bullet X_1\cdot\ldots\cdot X_{n-1}\cdot X_n, y_1 = \mathbf{x}] \in \langle\gamma\cdot X_n\cdot\ldots\cdot X_1\rangle_k$