SLR(k) and LALR(k) Parser

How many LR(k) states are there for LR parsers?

Upper bound: $2^{|R_k|}$.

$R_k = \{[A \rightarrow \alpha\bullet\beta, x] \mid A \rightarrow \alpha\beta \in P, x \in \Sigma^{\leq k}\}$

$|R_k| = |P| \times (|\Sigma|+1)^k$.

Too many states even for LR(1) parsers!!!

Let's consider LR(0) states

$R_0 = \{[A \rightarrow \alpha\bullet\beta] \mid A \rightarrow \alpha\beta \in P\}$

$|R_0| = |P|$.

Let $S \Rightarrow^*_{rm} \delta A z \Rightarrow_{rm} \delta\alpha\beta z \Rightarrow^*_{rm} \delta\alpha w z \Rightarrow^*_{rm} \delta v w z \Rightarrow^*_{rm} u v w z$.

We ***define*** $[A \rightarrow \alpha\bullet\beta]$ *be a valid LR(k) item for a viable* ***prefix*** $\delta\alpha$.

Let R_0 *denote a set of valid LR(0) items.*

We define $Valid_0 : (N \cup \Sigma)^* \rightarrow 2^{R_0}$ *or* $\langle \cdot \rangle_0$ *for short.*

$$Valid_0(\gamma) = \langle \gamma \rangle_0$$

$$= \{[A \rightarrow \alpha \bullet \beta] \in R_0 \mid S \Rightarrow^*_{rm} \delta A z \Rightarrow_{rm} \delta \alpha \beta z,\ \gamma = \delta \alpha\}$$

Lemma *If* $[A \rightarrow \alpha \bullet \beta \gamma] \in \langle \delta \alpha \rangle$, *then* $[A \rightarrow \alpha \beta \bullet \gamma] \in \langle \delta \alpha \beta \rangle_0$.

Proof

$$^{\exists} S \Rightarrow^*_{rm} \delta A z \Rightarrow_{rm} \delta \alpha \beta \gamma z.$$

$$\therefore\ [A \rightarrow \alpha \beta \bullet \gamma] \in \langle \delta \alpha \beta \rangle_0.$$

Cololary *If* $[A \rightarrow \alpha \bullet X \beta] \in \langle \delta \alpha \rangle_0$, $[A \rightarrow \alpha X \bullet \beta] \in \langle \delta \alpha X \rangle_0$.

Theorem *If* $[B \rightarrow \eta \bullet A \psi] \in \langle \delta \eta \rangle_0$ *and* $A \rightarrow \alpha \in P$, *then*

$[A \rightarrow \bullet \alpha] \in \langle \delta \eta \rangle_0$ *and vice versa.*

Proof $^{\exists} S \Rightarrow^*_{rm} \delta B u \Rightarrow_{rm} \delta \eta A \underline{\psi} u \Rightarrow^*_{rm} \delta \eta A \underline{v} u \Rightarrow_{rm} \delta \eta \alpha \beta v u = \gamma \alpha \beta v u,$

Let $K \subseteq R_0$.

Definition $\partial_{LR(0)}: 2^{R_0} \rightarrow 2^{R_0}$, ∂_0 *or* ∂ *for short.*

(**desc**$_{LR(0)}$, **desc**$_0$ *in text).*

$$\partial_0 K = \{[B \rightarrow \bullet\eta, y] \in R_0 \mid [A \rightarrow \alpha\bullet B\beta, x] \in K, B \rightarrow \eta \in P, y \in First_k(\beta \cdot x)\}$$

Definition $\chi_{LR(0)}{}^X: 2^{R_0} \times (N \cup \Sigma) \rightarrow 2^{R_0}$, $\chi_0{}^X$ *or* χ^X *for short*

(**passes-X**$_{LR(0)}$, **passes-X** *in text).*

$$\chi_0{}^X K = \{[A \rightarrow \alpha X\bullet\beta, x] \in R_0 \mid [A \rightarrow \alpha\bullet X\beta, x] \in K\}$$

If $K \in \langle\delta\rangle_0$, $\partial_0{}^* K \in \langle\delta\rangle_0$.

If $K \in \langle\delta\rangle_0$, $\chi_0{}^X K \in \langle\delta \cdot X\rangle_0$.

We define $\rho_{LR(0)} \subseteq (N \cup \Sigma)^* \times (N \cup \Sigma)^*$ ρ_0 *or* ρ *for short.*

$\gamma\ \rho_0\ \delta$, *if* $\langle\gamma\rangle_0 = \langle\delta\rangle_0$.

ρ_0 *is an* ***equivalent*** *binary relation on* $(N \cup \Sigma)^*$.

$[\gamma]_0 = \{\delta \in (N \cup \Sigma)^* \mid \gamma\ \rho_0\ \delta\}$ *equivalent class on* $(N \cup \Sigma)^*$.

We extend the domain of $\langle\cdot\rangle_0$ *from* $(N \cup \Sigma)^*$ *to* $2^{(N \cup \Sigma)^*}$.

We may write $\langle\delta\rangle_0$ *instead of* $\langle[\delta]_0\rangle_0$. *since* $\langle\delta\rangle_0 = \langle[\delta]_0\rangle_0$.

We may write $\langle\delta\rangle_0$ *instead of* $[\delta]_0$, *or vice versa, since*

$\langle\delta\rangle_0 \leftrightarrow^{1:1} [\delta]_0$.

Canonical Collection of <u>*LR(0) states*</u>*,* C_0*, and* $Q: C_k \times (N \cup \Sigma) \rightarrow C_k$.

(<u>*sets of LR(0) items*</u>, <u>*equivalent classes of valid viable prefices*</u>).

$\langle \varepsilon \rangle_0 := \partial_0^{*}([S' \rightarrow \bullet S]);\ C_0 := \{\langle \varepsilon \rangle_0\};\ P := \emptyset;$

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for $\langle \delta \rangle_0 \in C_0$ ***do***

for $X \in N \cup \Sigma$ ***where*** $K^X = \{[A \rightarrow \alpha \bullet X \beta] \in \langle \delta \rangle_0\}$ ***do***

$\langle \delta \cdot X \rangle_0 := \partial_0^{*}(\chi_0^X(K^X));$

$C_0 := C_0 \cup \langle \delta \cdot X \rangle_0;$

$P := P \cup \{\langle \delta \rangle_0 \cdot X \rightarrow \langle \delta \cdot X \rangle_0\}$

od ***od***

until *no more states are added to* C_0.

Fact & Construction of *shift actions of* Γ *in LR(0) parsing.*

If $[B \to \alpha \bullet \boldsymbol{X} \beta] \in \langle\delta\rangle_0 \in C_0$ *and* $\boldsymbol{X} \in N \cup \Sigma$ *, then*

$^{\exists}[B \to \alpha \boldsymbol{X} \bullet \beta] \in \langle\delta\cdot\boldsymbol{X}\rangle_0 \in C_0$ *and*

$$\textbf{Add } \langle\delta\rangle_0 \mid \boldsymbol{X} \to \langle\delta\rangle_0\langle\delta\cdot\boldsymbol{X}\rangle_0 \mid \in \Gamma.$$

Fact & Construction of *reduce actions of* Γ *in LR(0) parsing.*

If $[B \to \alpha \bullet A \beta] \in \langle\delta\rangle_0 \in C_0$ *and* $A \to X_1 \cdot X_2 \cdot \ldots \cdot X_n \in P$*, then*

$^{\exists}[A \to \bullet X_1 \cdot X_2 \cdot \ldots \cdot X_n] \in \langle\delta\rangle_0$,

$[1 \le {}^{\forall} i \le n: {}^{\exists}[A \to X_1 \cdot X_2 \ldots \cdot X_i \bullet X_{i+1} \cdot \ldots \cdot X_n] \in \langle\delta X_1 \cdot X_2 \cdot \ldots \cdot X_i\rangle_0,$

$^{\exists}\langle\delta\cdot X_1 \cdot \ldots \cdot X_{i-1}\rangle_0 \mid X_i \to \langle\delta\cdot X_1 \cdot \ldots \cdot X_{i-1}\rangle_0\langle\delta\cdot X_1 \cdot \ldots \cdot X_{i-1} \cdot X_i\rangle_0 \mid \in \Gamma],$

$^{\exists}[B \to \alpha A \bullet \beta] \in \langle\delta\cdot A\rangle_0 \in C_0, \langle\delta\rangle_0 \mid \to \langle\delta\rangle_0\langle\delta\cdot A\rangle_0 \mid \in \Gamma,$

$$\textbf{Add } \langle\delta\rangle_0\langle\delta\cdot X_1\rangle_0\langle\delta\cdot X_1 \cdot X_2\rangle_0 \ldots \langle\delta\cdot X_1 \cdot X_2 \ldots X_n\rangle_0 \mid \to \langle\delta\rangle_0\langle\delta\cdot A\rangle_0 \mid \in \Gamma.$$

LR(0) *parser is a* ***pdt*** $M_0 = \{C_0, \Sigma, \Gamma, P, \tau, [\varepsilon]_0, \{[\varepsilon]_0[S]_0\}, \$, \mid)$ *where*

$$\Gamma = \{[\delta]_0[\delta\cdot X_1]_0[\delta\cdot X_1\cdot X_2]\ldots[\delta\cdot X_1\cdot X_2\cdot\ldots\cdot X_n]_0 \mid \to [\delta]_0[\delta\cdot \mathbf{A}]_0 \mid \mid$$
$$[\mathbf{A} \to X_1X_2\ldots X_n\bullet] \in \langle\delta\cdot X_1X_2\ldots X_n\rangle_0\}$$
$$\cup \{[\delta]_0 \mid \mathbf{a}x \to [\delta]_0[\delta\cdot\mathbf{a}]_0 \mid x \mid [A \to \alpha\bullet\mathbf{a}\beta, y] \in \langle\delta\rangle_0\}$$

$$\tau([\delta]_0[\delta\cdot X_1]_0\ldots[\delta\cdot X_1X_2\cdot\ldots\cdot X_n]_0 \mid \to [\delta]_0[\delta\cdot\mathbf{A}]_0 \mid) = \mathbf{A} \to X_1\ldots X_n \in P,$$
$$\tau([\delta]_0 \mid \mathbf{a}x \to [\delta]_0[\delta\cdot\mathbf{a}]_0 \mid x) = \varepsilon.$$

$[B \to \alpha\bullet\mathbf{A}\beta, y] \in \langle\delta\rangle_0$ $\qquad$ $[B \to \alpha\mathbf{A}\bullet\beta] \in \langle\delta\cdot\mathbf{A}\rangle_0$

$[A \to \bullet X_1X_2\ldots X_n] \in \langle\delta\rangle_0$

$[A \to X_1\bullet X_2\ldots X_n] \in \langle\delta\cdot X_1\rangle_0$ $\qquad$ $[A \to \alpha\bullet\mathbf{a}\beta] \in \langle\delta\rangle_0$

$[A \to X_1X_2\bullet\ldots X_n] \in \langle\delta\cdot X_1\cdot X_2\rangle_0$ $\qquad$ $[A \to \alpha\mathbf{a}\bullet\beta] \in \langle\delta\cdot\mathbf{a}\rangle_0$

…

$[A \to X_1X_2\ldots X_n\bullet] \in \langle\delta\cdot X_1\ldots X_n\rangle_0$

[DeRemer FL(1971), Simple LR(k) grammars, Comm. ACM 14: pp453 - 460.

LR(0) grammars are seldom.
Reduce on Follow(A)

Simple LR(k) or SLR(k) for short.

SLR(k) *parser is a* ***pdt*** $M_0^k = \{C_0, \Sigma, \Gamma, P, \tau, [\varepsilon]_0, \{[\varepsilon]_0[S]_0\}, \$, \mid)$ *where*

$$\Gamma = \{[\delta]_0[\delta\cdot X_1]_0[\delta\cdot X_1\cdot X_2]\ldots[\delta\cdot X_1\cdot X_2\cdot\ldots\cdot X_n]_0 \mid \mathbf{x} \to [\delta]_0[\delta\cdot \mathbf{A}]_0 \mid \mathbf{x} \mid$$

$$[\mathbf{A} \to X_1X_2\ldots X_n\bullet] \in \langle\delta\cdot X_1X_2\ldots X_n\rangle_0,\ \mathbf{x} \in Follow_k(\mathbf{A})\}$$

$$\cup\ \{[\delta]_0 \mid \mathbf{a}x \to [\delta]_0[\delta\cdot\mathbf{a}]_0 \mid x \mid [A \to \alpha\bullet\mathbf{a}\beta, y] \in \langle\delta\rangle_0\}$$

$$\tau([\delta]_0[\delta\cdot X_1]_0\ldots[\delta\cdot X_1X_2\cdot\ldots\cdot X_n]_0 \mid \mathbf{x} \to [\delta]_0[\delta\cdot\mathbf{A}]_0 \mid \mathbf{x}) = \mathbf{A} \to X_1\ldots X_n \in P,$$

$$\tau([\delta]_0 \mid \mathbf{a}x \to [\delta]_0[\delta\cdot\mathbf{a}]_0 \mid x) = \varepsilon.$$

$[B \to \alpha\bullet\mathbf{A}\beta, y] \in \langle\delta\rangle_0$ $\qquad$ $[B \to \alpha\mathbf{A}\bullet\beta] \in \langle\delta\cdot\mathbf{A}\rangle_0$

$[A \to \bullet X_1X_2\ldots X_n] \in \langle\delta\rangle_0$

$[A \to X_1\bullet X_2\ldots X_n] \in \langle\delta\cdot X_1\rangle_0$ $\qquad$ $[A \to \alpha\bullet\mathbf{a}\beta] \in \langle\delta\rangle_0$

$[A \to X_1X_2\bullet\ldots X_n] \in \langle\delta\cdot X_1\cdot X_2\rangle_0$ $\qquad$ $[A \to \alpha\mathbf{a}\bullet\beta] \in \langle\delta\cdot\mathbf{a}\rangle_0$

…

$[A \to X_1X_2\ldots X_n\bullet] \in \langle\delta\cdot X_1\ldots X_n\rangle_0$

LALR(k) *parser is a* **pdt** $M_0^k = \{C_0, \Sigma, \Gamma, P, \tau, [\varepsilon]_0, \{[\varepsilon]_0[S]_0\}, \$, \mid)$ *where*

$\Gamma = \{[\delta]_0[\delta\cdot X_1]_0[\delta\cdot X_1\cdot X_2]\ldots[\delta\cdot X_1\cdot X_2\cdot\ldots\cdot X_n]_0 \mid \mathbf{x} \to [\delta]_0[\delta\cdot \mathbf{A}]_0 \mid \mathbf{x} \mid$

$[\mathbf{A} \to X_1X_2\ldots X_n\bullet, \mathbf{x}] \in \langle[\delta\cdot X_1X_2\ldots X_n]_0\rangle_k\}$

$\cup \{[\delta]_0 \mid \mathbf{a}x \to [\delta]_0[\delta\cdot \mathbf{a}]_0 \mid x \mid [A \to \alpha\bullet\mathbf{a}\beta, y] \in \langle\delta\rangle_0\}$

$\tau([\delta]_0[\delta\cdot X_1]_0\ldots[\delta\cdot X_1X_2\cdot\ldots\cdot X_n]_0 \mid \mathbf{x} \to [\delta]_0[\delta\cdot \mathbf{A}]_0 \mid \mathbf{x})$

$= \mathbf{A} \to X_1\ldots X_n \in P,$

$\tau([\delta]_0 \mid \mathbf{a}x \to [\delta]_0[\delta\cdot \mathbf{a}]_0 \mid x) = \varepsilon.$

$[B \to \alpha\bullet\mathbf{A}\beta] \in \langle\delta\rangle_0$ $\quad$ $[B \to \alpha\mathbf{A}\bullet\beta] \in \langle\delta\cdot\mathbf{A}\rangle_0$

$[A \to \bullet X_1X_2\ldots X_n] \in \langle\delta\rangle_0$

$[A \to X_1\bullet X_2\ldots X_n] \in \langle\delta\cdot X_1\rangle_0$ $\quad$ $[A \to \alpha\bullet\mathbf{a}\beta] \in \langle\delta\rangle_0$

$[A \to X_1X_2\bullet\ldots X_n] \in \langle\delta\cdot X_1\cdot X_2\rangle_0$ $\quad$ $[A \to \alpha\mathbf{a}\bullet\beta] \in \langle\delta\cdot\mathbf{a}\rangle_0$

…

$[A \to X_1X_2\ldots X_n\bullet] \in \langle\delta\cdot X_1\ldots X_n\rangle_0$

Computation of LALR(k) lookahead sets.

*[DeRemer FL, **Efficient computation of LALR(k) lookahead sets**, ACM Tran. on Programming Languages and Systems, 4, pp615-649, 1982]*

Definition *Let* $[A \to \alpha\bullet\beta] \in \langle\delta\cdot\alpha\rangle_0$. *Then* $Lalr_k: R_0 \times C_0 \to 2^{\Sigma^{\leq k}}$:

$$Lalr_k([A \to \alpha\bullet\beta], \langle\delta\cdot\alpha\rangle_0) = \{\mathbf{x} \in \Sigma^{\leq k} \mid [A \to \alpha\bullet\beta, \mathbf{x}] \in \langle[\delta\cdot\alpha]_0\rangle_{\mathbf{k}}\}$$

Lemma *Let* $[A \to \alpha\bullet\beta] \in \langle\delta\cdot\alpha\rangle_0$. *Then*

$$Lalr_k([A \to \alpha\bullet\beta], \langle\delta\cdot\alpha\rangle_0)$$
$$= Lalr_k([A \to \alpha\beta\bullet], \langle\delta\cdot\alpha\cdot\beta\rangle_0) = Lalr_k([A \to \bullet\alpha\beta], \langle\delta\rangle_0)$$

where $[A \to \alpha\beta\bullet] \in \langle\delta\cdot\alpha\cdot\beta\rangle_0$ *and* $[A \to \bullet\alpha\beta] \in \langle\delta\rangle_0$.

Theorem *Let* $[A \to \alpha\bullet\beta] \in \langle\delta\cdot\alpha\rangle_0$. *Then*

$$Lalr_k([A \to \alpha\bullet\beta], \langle\delta\cdot\alpha\rangle_0) = Lalr_k([A \to \bullet\alpha\beta], \langle\delta\rangle_0)$$
$$= First_k(\psi\cdot Lalr_k([B \to \eta\bullet A\psi], \langle\delta\rangle_0))$$
$$= First_k(\psi) \oplus_k Lalr_k([B \to \eta\bullet A\psi], \langle\delta\rangle_0).$$
$$\text{where } [B \to \eta\bullet A\psi] \in \langle\delta\rangle_0.$$

Recursive formula *for* $Lalr_k$.

recursion

$$Lalr_k([A \to \alpha\bullet\beta], \langle\delta\cdot\alpha\rangle_0) = First_k(\psi) \oplus_k Lalr_k([B \to \eta\bullet A\psi], \langle\delta\rangle_0).$$

basis

$$Lalr_k([S' \to \bullet S], \langle\varepsilon\rangle_0) = \{\varepsilon\}.$$

Choe's Ph. D. Thesis

Park, CH, Choe KM, Chang CH(1985), ***A new analysis of LALR(k) formalisms****, ACM Tran. on Programming Languages and Systems, 7, pp159-175.*

Observation Construcion *of* C_k *and* ***computation*** *Lalr*$_k$.

Recomputations of ∂_k^* *in every same LR(k) state are* ***redundant****!*

Kernel(Essential *in text) items are enough to consider.*

Definition essential *LR(k) items.*

$[A \rightarrow \alpha \bullet \beta, x] \in R_k$ *is a* ***essential*** *LR(k) item, if* $\alpha \neq \varepsilon$ *or* $A = S'$.

Definition *Let* $\langle\langle\delta\rangle\rangle_k \in C_k$ *denote a set of valid* ***essential*** *items in* $\langle\delta\rangle_k$.

Then $\langle\langle\delta\rangle\rangle_k \subseteq \langle\delta\rangle_k$ *and* $\partial_k^* \langle\langle\delta\rangle\rangle_k = \langle\delta\rangle_k = \partial_k^* \langle\delta\rangle_k$.

$$\therefore \langle\delta\rangle_k = \partial_k^{*}\langle\!\langle\delta\rangle\!\rangle_k = \langle\!\langle\delta\rangle\!\rangle_k \cup \partial_k^{+}\langle\!\langle\delta\rangle\!\rangle_k.$$

where $\langle\!\langle\delta\rangle\!\rangle_k \cap \partial_k^{+}\langle\!\langle\delta\rangle\!\rangle_k = \varnothing.$

$\langle\!\langle\delta\rangle\!\rangle_k$ *essential items*

$\partial_k^{+}\langle\!\langle\delta\rangle\!\rangle_k$ *non-essential items*

$$\therefore \langle\!\langle\delta\rangle\!\rangle_k \leftrightarrow^{1:1} \langle\delta\rangle_k \leftrightarrow^{1:1} [\delta]_k.$$

The following statements are equivalent:

An LR(k) state	$\langle\!\langle\delta\rangle\!\rangle_k \leftrightarrow^{1:1} \langle\delta\rangle_k \leftrightarrow^{1:1} [\delta]_k.$
A set of valid LR(k) items	$\langle\delta\rangle_k.$
*A set of valid **essential** LR(k) items*	$\langle\!\langle\delta\rangle\!\rangle_k.$
A set of valid viable prefices	$[\delta]_k.$
A set of valid stack strings	$[\delta]_k.$

Fact *Let* $[A \to \alpha \bullet X\beta, x] \in \langle\delta\rangle_k$. *Then* $\chi_k^X([A \to \alpha \bullet X\beta, x]) \in \langle\langle\delta\cdot X\rangle\rangle_k$.

Observation *All iems in* $\chi_0^X(K^X)$ *and* $[S' \to \bullet S]$ *in algorithm for of* C_0 *in page 5 are* ***essential*** *but* K^X *may contain* ***non-essential*** *items.*

Definition *L relation,* $L \subseteq N \times (N \cup \Sigma)$.
$A\ L\ X$, *if* $A \to X\beta \in P$.

Lemma *Let* $\langle\delta\rangle_k \in C_k$, $^{\exists}[A \to \alpha \bullet X\beta] \in \langle\langle\delta\rangle\rangle_k (\alpha \neq \varepsilon$ *or* $A = S')$. *Then* $^{\exists}Y \in N \cup \Sigma$.э. $X\ L^{*}Y$ *and* $\chi_k^Y(\langle\langle\delta\rangle\rangle_k) \in \langle\langle\delta\cdot Y\rangle\rangle_k$.

Essential items and L relation are enough for the ***definition*** *and the* ***construction*** *of not only* C_0 *but* C_k.

Canonical Collection of LR(k) states, C_k,

*(sets of **essential** LR(k) items, equivalent classes of valid viable prefices).*

$\langle\!\langle \varepsilon \rangle\!\rangle_k := \{[S' \to \bullet S]\}$; $C_0 := \{\langle\!\langle \varepsilon \rangle\!\rangle_k\}$; $P := \emptyset$;

repaeat

  for $\langle\!\langle \delta \rangle\!\rangle_k \in C_k$ ***do***

    for $X \in N \cup \Sigma$ ***where*** $K^X = \{[B \to \eta \bullet X \psi] \in \langle\!\langle \delta \rangle\!\rangle_k$ ***do***

      for $Y \in N \cup \Sigma$ ***where*** $X\, L^* Y$, $K^Y = \{[A \to \alpha \bullet Y \beta] \in \langle \delta \rangle_k\}$ ***do***

        $\langle\!\langle \delta \cdot Y \rangle\!\rangle_k := \chi_k{}^Y(K^Y)$;

        $C_k := C_k \cup \langle\!\langle \delta \cdot Y \rangle\!\rangle_k$;

        $P := P \cup \{\langle\!\langle \delta \rangle\!\rangle_k \cdot Y \to \langle\!\langle \delta \cdot Y \rangle\!\rangle_k\}$

  od   ***od***   ***od***

until *no more states are added to* C_k.

Lemma *If* $[C \to \eta \bullet B\psi, y] \in \langle\!\langle\delta\rangle\!\rangle_k$ *and* $^{\exists}A \in N$.э. $B\ L^n\ A$ *where*

$B = B_0$, $[1 \le {}^{\forall}i \le n: {}^{\exists}B_{i-1} \to B_i\gamma_i \in P]$, $B_n = A$, *and* $A \to \alpha\beta \in P$. *Then*

$[1 \le {}^{\forall}i \le n: {}^{\exists}[B_{i-1} \to \bullet B_i\gamma_i, y_i] \in \partial_k{}^{+}\langle\!\langle\delta\rangle\!\rangle_k$

where $y_i \in First_k(\gamma_{i-1}\ldots\gamma_1 \cdot \psi \cdot y)$ *if* $i=1$, $\gamma_{i-1}\ldots\gamma_1 = \varepsilon]$,

$[A \to \bullet\alpha\beta, \mathbf{x}] \in \partial_k{}^{+}\langle\!\langle\delta\rangle\!\rangle_k$ *where* $\mathbf{x} \in First_k(\gamma_n\ldots\gamma_1 \cdot \psi \cdot y)$, *and*

$[A \to \alpha\bullet\beta, \mathbf{x}] \in \langle\delta\cdot\alpha\rangle_k$.

$\therefore Lalr_k([A \to \alpha\bullet\beta], \langle\!\langle\delta\cdot\alpha\rangle\!\rangle_0)$.

$= First_k(\gamma_n) \oplus_k Lalr_k([B_{n-1} \to \bullet B_n\gamma_n], \underline{\langle\delta\rangle_0})$ $\qquad A = B_n$.

$= First_k(\gamma_n) \oplus_k First_k(\gamma_{n-1}) \oplus_k Lalr_k([B_{n-2} \to \bullet B_{n-1}\gamma_{n-1}], \underline{\langle\delta\rangle_0})$.

$= First_k(\gamma_n\gamma_{n-1}) \oplus_k First_k(\gamma_{n-2}) \oplus_k Lalr_k([B_{n-3} \to \bullet B_{n-2}\gamma_{n-2}], \underline{\langle\delta\rangle_0})$.

…

$= First_k(\gamma_n\ldots\gamma_2) \oplus_k First_k(\gamma_1) \oplus_k Lalr_k([B_0 \to \bullet B_1\gamma_1], \underline{\langle\delta\rangle_0})$.

$= First_k(\gamma_n\ldots\gamma_1) \oplus_k First_k(\psi) \oplus_k Lalr_k([C \to \eta\bullet B\psi], \underline{\langle\!\langle\delta\rangle\!\rangle_0})$ $\qquad B_0 = B$.

Definition *Let* $A, B \in N$, $A\ L\ B$, *and* $A \rightarrow B\beta \in P$. *Then*

we define **path**$_k$: $N \times N \rightarrow 2^{\Sigma^{\leq k}}$ *as* **path**$_k(A, B) = First_k(\beta)$.

Definition *Let* $A, B \in N$ *and* $A\ L^*\ B$. *Then we define* $Path_k: N \times N \rightarrow 2^{\Sigma^{\leq k}}$.

$Path_k(A, A) = \{\varepsilon\}$

$Path_k(A, B) = \mathbf{path}_k(C, B) \oplus_k Path_k(A, C)$, *if* $A\ L^+\ B$

where $\exists C \in N$.э. $A\ L^*\ C$ *and* $C\ L\ B$.

Essential items, L relation, and $Path_k$ *are enough*

for the ***definition****, the* ***construction*** *of* C_k,

and the ***computation*** *of* $Lalr_k$.

Example *What is the* $Path_k(B, A)$ *in the above* ***Lemma***.

Theorem *Let* $[A \rightarrow \alpha \bullet \beta] \in \langle\langle \delta \cdot \alpha \rangle\rangle_0$. *Then*

$$Lalr_k([A \rightarrow \alpha \bullet \beta], \langle\langle \delta \cdot \alpha \rangle\rangle_0)$$

$$= Path_k(A', A) \oplus_k First_k(\psi) \oplus_k Lalr_k([B \rightarrow \eta \bullet A'\psi], \langle\langle \delta \rangle\rangle_0).$$

$$\text{where } A' \, L^* \, A, \; [B \rightarrow \eta \bullet A'\psi] \in \langle\langle \delta \rangle\rangle_0.$$

Recursive formula *for* $Lalr_k$.

recursion

$$Lalr_k([A \rightarrow \alpha \bullet \beta], \langle\langle \delta \cdot \alpha \rangle\rangle_0)$$

$$Path_k(A', A) \oplus_k First_k(\psi) \oplus_k Lalr_k([B \rightarrow \eta \bullet A'\psi], \langle\langle \delta \rangle\rangle_0)$$

$$\text{where } A' \, L^* \, A, \; [B \rightarrow \eta \bullet A'\psi] \in \langle\langle \delta \rangle\rangle_0.$$

basis

$$Lalr_k([S' \rightarrow \bullet S], \langle\langle \varepsilon \rangle\rangle_0) = \{\varepsilon\}.$$