

algebraic system  $(A, \cdot)$  is closed ~~error~~ :  $A \times A \rightarrow A$  (binary operation)  
 Ex)  $(\mathbb{N}, +)$  is infinite computer integer is not closed!  
 pseudo real, pseudo integer

semi-group  $(A, \cdot)$   $\left\{ \begin{array}{l} \text{binary operation} \\ \text{error band} \end{array} \right.$   
 ① algebraic system ( $\cdot$  is closed)  
 ②  $\cdot$  is associative.  $\forall a, b, c \in A (a \cdot b) \cdot c = a \cdot (b \cdot c)$   
 $\cdot$  is n-ary operation

Ex)  $(\mathbb{N}, +) \sum_{i=1}^n a_i$

③ identity  $\forall a \in A, \exists e \in A a \cdot e = e \cdot a = a$   $e$ : identity  
 Ex)  $(\mathbb{N}, +, 0)$   $(\mathbb{N}, \times, 1)$  is a monoid.

$(\mathbb{N}, \cdot, e)$  is a monoid

Fact 1.12 - HW #2 prove fact 1.12 Due 3/22 (木) 9:00

$\left[ \begin{array}{l} x^0 \triangleq e \\ x^n \triangleq x \cdot x^{n-1} \quad (n \geq 1; n > 0) \end{array} \right.$  concatenation operator.

Formal Language Theory

Terminologies

From	set	
symbol	$a \in V$ (symbol)	$V$ (Vocabulary) - set of symbol alphabet
string	$x \in V^*$ (string) 文字열	$V^*$ (Universe of strings) $L \subseteq V^*$ (language) empty string.
		$\epsilon \notin V$ $\forall a \in V  a  = 1$ $\forall a \in V^* \text{ 한글 Voc. } V$ $V = \{ \epsilon, L, \dots \}$ , 14 $\{ \epsilon, F, \dots \}$ , 10 $\{ \epsilon, \alpha, \beta, \dots \}$ , 22 } 5 7월 7일 < 7월 22일 < 7월 29일

$(V^*, \cdot, \epsilon)$  is a free monoid

# 1.6 Rewriting Systems

$G = (V, P)$  is a rewriting system (or Semi Thue System)

$V$ : vocabulary

$$P \subseteq V^* \times V^*$$

$$\alpha, \beta \in V^* \quad (\alpha, \beta) \in P \text{ or } \alpha \rightarrow \beta \in P$$

$$u = \gamma \alpha \delta \xrightarrow[\substack{\text{written} \\ \text{in } G}]{\alpha \Rightarrow \beta} \gamma \beta \delta \quad \forall \gamma, \delta \in V^*$$

$u$  (directly) derive  $\gamma \beta \delta$ .

$$u_1 \Rightarrow u_2 \Rightarrow u_3 \Rightarrow \dots \Rightarrow u_n$$

$$u_1 \Rightarrow^n u_n \text{ or } u \Rightarrow^* u_n$$

or  $u \Rightarrow^\pi \xi$  where  $u, \xi \in V^*, \pi \in P^*$