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제4강 3/13(화) ~~(V^*, ε) mono. A~~, Cantor's Diagonal Argument.

Review: Relation, function, Cardinality of set

Cardinality of sets

1. finite $\{1, 2, \dots, n\}$
2. infinite $\{1, 2, \dots\}$

A set is countable (denumerable, enumerable) $\left\langle \begin{array}{l} \text{finite} \\ \text{countable infinite} \end{array} \right.$

$|A| = |B| \in \mathcal{P}(\mathbb{N})$

A set is uncountable, otherwise. ($\sqrt{2}$ r?)

Lemma 1.9 $\underbrace{\bigcup_{i \in \mathbb{N}_0} A_i}_{\text{countably infinite union of}} \to$ pairwise disjoint finite set $\Longrightarrow$ countably infinite

Let V be a set.

$V^0 = \{\varepsilon\}$

$V^n = V^{n-1} \cdot V$

$V^* = \bigcup_{i \in \mathbb{N}_0} V^i$

Project:
Simulator for
Dijkstra's Mini-language
Due: 종강 (6/14: 목) 전
6/7: 목 까지 밤 12:00

Let A and B be sets.

$A^B \triangleq \{f \mid f : B \to A\}$ $\quad |A^B| = |A|^{|B|}$

Ex. $A = V$, $B = \{1, 2, \dots, n\}$ $\quad f : \{1, 2, \dots, n\} \to V \triangleq V^n$ $\quad |V^N| = |V|^n$

$f(1), f(2), \dots, f(n) \in V$

$2^A = \mathcal{P}(A) \triangleq \{B \mid B \subseteq A\}$ $\quad |2^A| = 2^{|A|}$

Cantor Diagonal argument $|V^{\mathbb{N}}| = |2^{\mathbb{N}}|$ is uncountable!

Proof $\{0, 1\}^{\mathbb{N}}$ is uncountable

Assume $\{0, 1\}^{\mathbb{N}}$ is countable

$\{0, 1\}^n = \{f_0, f_1, \dots\}$

Consider complement of diagonal element: f

$f \notin \{f_0, f_1, \dots\}$ but $f \in \{0, 1\}^{\mathbb{N}}$

$\therefore \{0, 1\}^{\mathbb{N}}$ is countable is a contradiction.

$\therefore$ Q.E.D. $\quad \{0, 1\}^n = \{f_0, f_1, \dots\}$