

3/8 (\*) M33 binary relation, graph, equivalence rel. partition, partial order, function

$R \subseteq A \times A$   
 $G = (V, E)$   
 $V \times V \subseteq E$   
 $a, b \in V, aRb$



$aRb$   
 $aR^2b$   
 $aR^3b$

$aR^i b$   
 $aR^{i+1} b$   
 $\vdots$   
 $aR^+ b = \bigcup_{i \in \mathbb{N}_0} aR^i b$

$R^+ = \bigcup_{i \in \mathbb{N}_0} R^i$   
 where  $\mathbb{N}_0 = \{0, 1, 2, \dots\}$   
 $\mathbb{N}_1 = \{1, 2, 3, \dots\}$

Equi. rel. on A  $\leftrightarrow$  partition on A.

$\text{Par}(A) = \{A_1, A_2, \dots, A_k\}$   
 where  $\bigcup_{i=1}^k A_i = A$  and  $1 \leq i < j \leq k$   
 ① exhaustive  
 ② mutually disjoint.

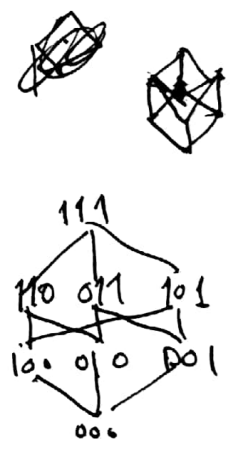
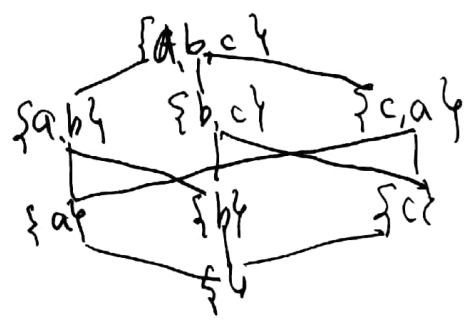
$|A| = |B| \Rightarrow A \cong B$

Equiv. rel  $\cong$  on  $\mathbb{N} \cong \mathbb{Z} \cong \mathbb{Z} \cong \mathbb{Z}$  partition  $\leftrightarrow \mathbb{N}$ .

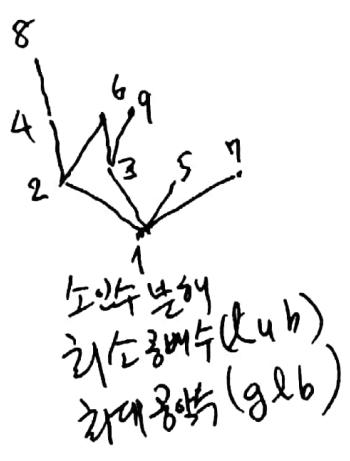
$R^*$ : reflexive-transitive closure of R  
 $R^+$ : transitive closure of R.

poset  $(A, \leq)$ , antisymmetric, transitive  
 (irreflexive)

Ex:  $(2^A, \subseteq)$  is a lattice poset.  
 $A = \{a, b, c\}$   
 $\exists$  lub, glb



$(\mathbb{N}_2, |)$  is a lattice



function  $f$  from  $A$  to  $B$

$$f: A \rightarrow B. \quad (f \subseteq A \times B \quad \forall a \in A, \exists! f(a) \in B)$$

① ~~uniqueness~~ total  
② uniqueness

$$|A| \begin{matrix} < \\ \leq \\ > \end{matrix} |B|$$

1-1  $|A| \leq |B|$

onto  $|A| \geq |B|$

1-1 & onto  $|A| = |B|$  —  $A \cong_f B$

bijection  $f: A \xleftrightarrow[onto]{1-1} B$