

Let  $G = (N, T, P, S)$  be a cfg.

**Def.**  $[A \rightarrow \alpha \bullet \beta]$  is called as an LR(0) item, if  $A \rightarrow \alpha \beta \in P$ .

Let  $I_G = \{[A \rightarrow \alpha \bullet \beta] \mid A \rightarrow \alpha \beta \in P\}$  be set of LR(0) items of  $G$ .

**Def.** An *augmented* grammar of  $G = (N, T, P, S)$  is

$$G' = (N \cup \{S'\}, T, P \cup \{S' \rightarrow S\}, S') \text{ where } S' \notin N.$$

**Fact,**  $L(G) = L(G')$

**Def.** The *rule automaton* for an augmented cfg  $G = (N, T, P, S)$  is an  $\varepsilon$ -NFA  $R_G = (I_G, N \cup T, \rightarrow_R, [S' \rightarrow \bullet S], \{[S' \rightarrow S \bullet]\})$  where

$$([A \rightarrow \alpha \bullet X \gamma], X) \rightarrow_R ([A \rightarrow \alpha X \bullet \gamma], \varepsilon) \text{ for } X \in N \cup T, \alpha, \gamma \in (N \cup T)^*,$$

$$([A \rightarrow \alpha \bullet B \gamma], \varepsilon) \rightarrow_R ([B \rightarrow \bullet \beta], \varepsilon) \text{ for } B \in N, \alpha, \gamma \in (N \cup T)^* B \rightarrow \beta \in P.$$

**Def.** A DFA of the  $\varepsilon$ -NFA  $R_G$  is  $D_G = (2^{I_G}, N \cup T, \rightarrow_D, \varepsilon^*([S' \rightarrow \bullet S]), \varepsilon^*([S' \rightarrow S \bullet]))$  where  $(\varepsilon^*([A \rightarrow \alpha \bullet X \gamma]), X) \rightarrow_D (\varepsilon^*([A \rightarrow \alpha X \bullet \gamma]), \varepsilon)$ .

**Alg. Collection of set of LR(0) items (LR(0) states):**  $C_0 \subseteq 2^I_G$ .

$C_0 := \{\varepsilon^*([S' \rightarrow \bullet S])\} /* = [\varepsilon] */$

**repeat**

**for**  $q \in C_0$  **do**

**for**  $\{[A \rightarrow \alpha \bullet X \gamma]\} \in q$  **do** where  $X \in N \cup T /* q = [\alpha^R \delta^R] */$

$p := \varepsilon^*([A \rightarrow \alpha X \bullet \gamma]) /* p = [X \alpha^R \delta^R] */$

**if**  $X \in T \rightarrow \text{shift}(q, X) = p$  |  $X \in N \rightarrow \text{goto}(q, X) = p$  **fi**

$C_0 := C_0 \cup \{q\}$

**od od**

**until**  $C_0$  does not increase

*Kernel and non kernel items of in LR(0) states  $p \in C_0$*

$K_p$ : Kernel items       $[A \rightarrow \alpha \bullet \beta] \in p$ , if  $(A = S') \vee (\alpha \neq \varepsilon)$ .

$p = \varepsilon^*(K_p) = K_p \cup \varepsilon^+(K_p)$ .      *Kernel items  $\cup$  Non kernel items*

Two actions in LR(0) right parser(PDA) with configuration  $T^* \times C_0^*$ .

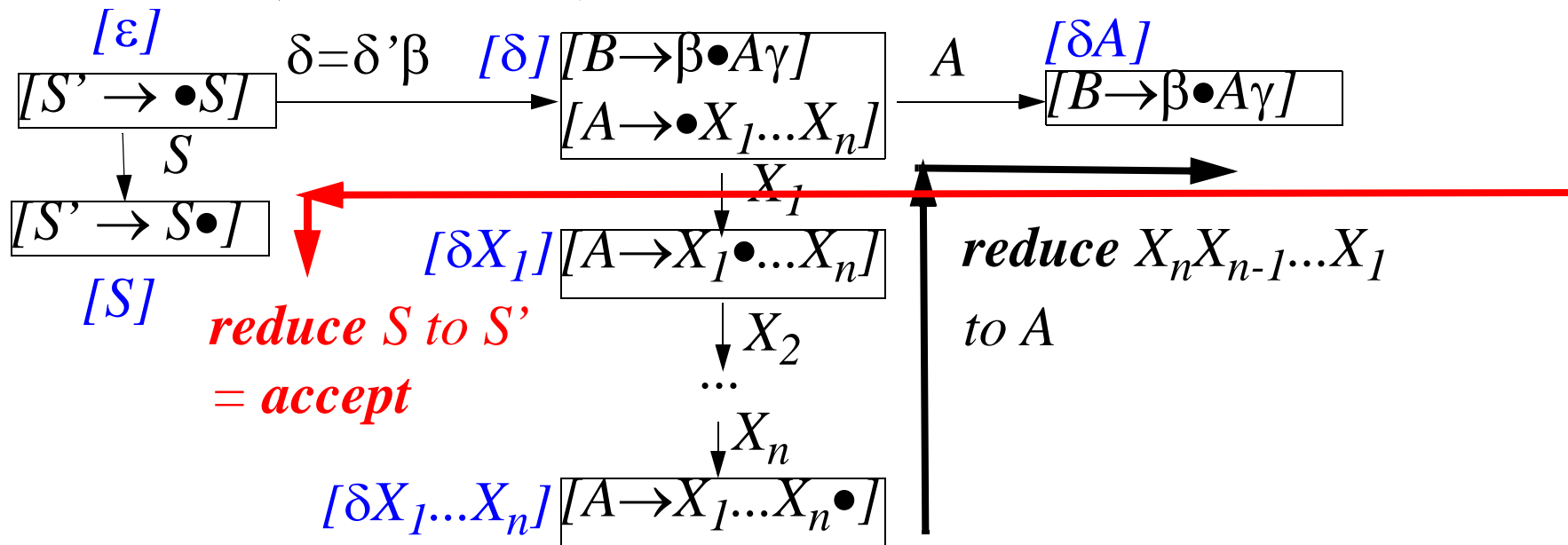
1) **shift**  $a$ :  $a \in T \quad [A \rightarrow \alpha \bullet a \beta] \in [\alpha^R \delta^R]$ .

$$(a, [\delta^R \alpha^R]) \rightarrow (a, [a \alpha^R \delta^R][\alpha^R \delta^R])$$

2) **reduce**  $\alpha$  to  $A$ :  $A \rightarrow \alpha \in P \quad [A \rightarrow \alpha \bullet] \in [\alpha^R \delta^R]$  where  $\alpha = X_1 X_2 \dots X_n$ .

$$(\epsilon, [X_n X_{n-1} \dots X_1 \delta^R][X_{n-1} \dots X_1 \delta^R] \dots [X_2 X_1 \delta^R][X_1 \delta^R][\delta^R])$$

$$\rightarrow (\epsilon, [\delta^R A][\delta^R])$$



**initial configuration**  $\iota = (x, [\epsilon])$

**final configuration**  $\Phi = \{(\epsilon, [\epsilon][S])\}$

**LR(0) parser** for  $G = (N, T, P, S)$ ,

$$P_{LR(0)} = (T^* \times C_0^*, \rightarrow_{LR(0)}, (x, [\epsilon]), (\epsilon, [\epsilon][S]))$$

**is deterministic, the  $G$  is LR(0) grammar.**

**Strong LR( $k$ ) (SLL( $k$ )) parser**

**SLR( $k$ ) parser**  $P_{SLR(k)} = (T^* \times C_0^*, \rightarrow_{SLR(k)}, (x, [\epsilon]), (\epsilon, [\epsilon][S]))$  is

1) **shift**  $a$ :  $a \in T$   $[A \rightarrow \alpha \bullet a \beta] \in [\alpha^R \delta^R]$ .

$$(a, [\delta^R \alpha^R]) \rightarrow (a, [a \alpha^R \delta^R][\alpha^R \delta^R])$$

2) **reduce**  $\alpha$  to  $A$ :  $A \rightarrow \alpha \in P$   $[A \rightarrow \alpha \bullet] \in [\alpha^R \delta^R]$  where  $\alpha = X_1 X_2 \dots X_n$ .

$$(x, [X_n X_{n-1} \dots X_1 \delta^R][X_{n-1} \dots X_1 \delta^R] \dots [X_2 X_1 \delta^R][X_1 \delta^R][\delta^R])$$

$$\rightarrow (x, [\delta^R A [\delta^R]]) \quad x \in \text{Follow}_k(A)$$

**Def.** LR( $k$ ) item:  $[A \rightarrow \alpha \bullet \beta, \mathbf{x}]$ , if  $A \rightarrow \alpha \beta \in P$  and  $x \in T^{\leq k}$ .

$\mathbf{x}$  is called the lookahead of the LR( $k$ ) item  $[A \rightarrow \alpha \bullet \beta, \mathbf{x}]$ .

**Alg.** Collection of set of LR( $k$ ) items (LR( $k$ ) states):  $C_k \subseteq 2^{I_G}$ .

$C_k := \{\varepsilon_k^*([S' \rightarrow \bullet S, \varepsilon])\} /* [\varepsilon] */$

where  $\varepsilon_k([A \rightarrow \alpha' \bullet B \gamma', \mathbf{x}]) = [B \rightarrow \bullet \beta, \mathbf{y}]$ ,  $\mathbf{y} \in \text{First}_k(\gamma' \mathbf{x})$

**repeat**

**for**  $q \in C_k$  **do**

**for**  $\{[A \rightarrow \alpha \bullet X \gamma, \mathbf{x}]\} \in q$  **do** where  $X \in N \cup T /* q = [\alpha^R \delta^R] */$

$p := \varepsilon_k^*([A \rightarrow \alpha X \bullet \gamma, \mathbf{x}]); /* p = [X \alpha^R \delta^R] */$

**if**  $X \in T \rightarrow \text{shift}(q, X) = p$  |  $X \in N \rightarrow \text{goto}(q, X) = p$  **fi**

$C_k := C_k \cup \{q\}$

**od od**

**until**  $C_k$  does not increase

**LR(k) parser** for  $G = (N, T, P, S)$  is

$P_{LR(k)} = (T^* \times C_k^*, \rightarrow_{LR(k)}, (x, [\varepsilon]_k), (\varepsilon, [\varepsilon]_k[S]_k))$  where

- 1) **shift**  $a$ :  $a \in T$   $[A \rightarrow \alpha \bullet a \beta, ax] \in [\alpha^R \delta^R], ax \in T^{\leq k}$ .  
 $(a, [\delta^R \alpha^R]) \rightarrow (a, [a \alpha^R \delta^R][\alpha^R \delta^R])$
- 2) **reduce**  $\alpha$  to  $A$ :  $A \rightarrow \alpha \in P$   $[A \rightarrow \alpha \bullet, x] \in [\alpha^R \delta^R]$  where  $\alpha = X_1 X_2 \dots X_n$ .  
 $(x, [X_n X_{n-1} \dots X_1 \delta^R][X_{n-1} \dots X_1 \delta^R] \dots [X_2 X_1 \delta^R][X_1 \delta^R][\delta^R])$   
 $\rightarrow (x, [\delta^R A [\delta^R]])$

If LR(k) parser for  $G$ ,  $P_{LR(k)}$ , is deterministic  $G$  is called **LR(k) grammar**.

**Lookahead LR(k) parser** (**LALR(k) = LR(0, k) parser**)

state: LR(0) states

reduce: LR(k) lookahead