6-B Deterministic Left Parsers

Let G = (N, T, P, S) is a **context-free grammar**. *A* **non-deterministic Left Parser** $L_P = ((T^* \times V^*), \rightarrow_L, (x, S), \{(\varepsilon, \varepsilon)\}, P, \tau)$ *with*

$$\rightarrow_L = \{(\varepsilon, A) \rightarrow_L^{A\rightarrow\alpha} (\varepsilon, \alpha) \in (T^* \times V^*)^2 | (A \rightarrow \alpha) \in P\}$$

guess *A* **as** α: **non-deterministic**.

$$\cup \{(a, a) \rightarrow_L^{a} (\varepsilon, \varepsilon) \in (T^* \times V^*)^2 | a \in \Sigma\}$$

verify $a \in \Sigma$: **deterministic**.

Consider **two guess** *actions* $B \rightarrow \beta \mid \beta'$ *where* $\beta \neq \beta'$.

$(\varepsilon, B) \rightarrow_L^{B\rightarrow\beta} (\varepsilon, \beta)$ *or* $(\varepsilon, B) \rightarrow_L^{B\rightarrow\beta'} (\varepsilon, \beta')$ *is* **non-deterministic**.

Consider $L(\beta)$ *and* $L(\beta')$!

Def. $L: (N \cup T)^* \rightarrow 2^{T^*}$. $L(\alpha) = \{x \in T^* | \alpha \Rightarrow^* x, x \in T^*\}$ **infinite**.

But $L(\beta)$ *and* $L(\beta')$ *may be* **infinite**.

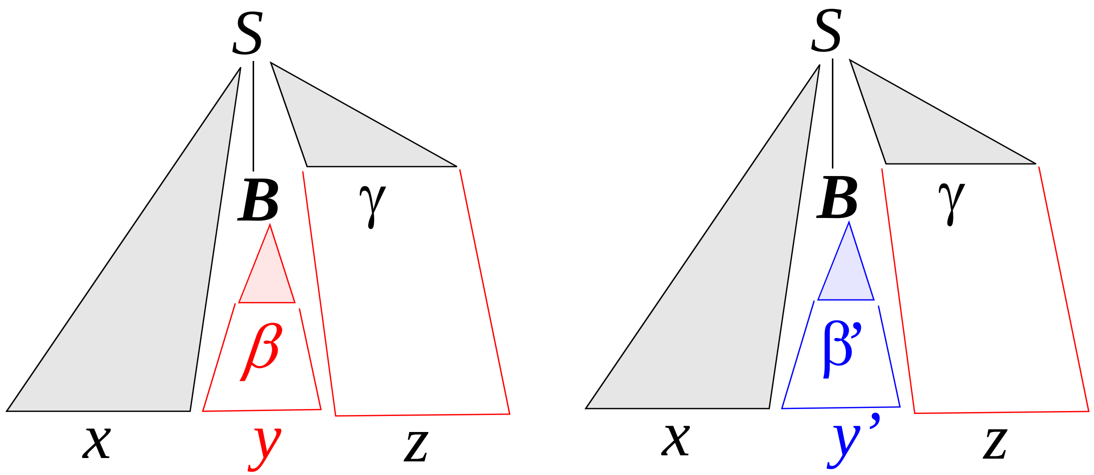
$\therefore\ First_k: (N \cup T)^* \rightarrow 2^{T^{\leq k}}$ where $T^{\leq k} = \{\varepsilon\} \cup T^1 \cup T^2 \cup \ldots \cup T^k$.

$First_k(\alpha) = k{:}L(\alpha) = \{k{:}x \in T^{\leq k} \mid \alpha \Rightarrow^* x,\ x \in T^*\}$ ***finite***.

If $First_k(\beta) \cap First_k(\beta') = \varnothing$, then lookahead string y and y'

$(y, B) \rightarrow_L (y, \beta),\ y \in First_k(\beta)$ and $(y', B) \rightarrow_L (y', \beta'),\ y' \in First_k(\beta')$

$\therefore$ ***deterministic***!



Computation *of* $First_k(\alpha)$ *for* $\alpha \in (N \cup T)^*$.

$First_k(\alpha) = First_k(X_1X_2...X_n)$ *where* $\alpha = X_1X_2...X_n$.

$= First_k(X_1) \oplus_k First_k(X_2) \oplus_k ... \oplus_k First_k(X_n)$ *where*

$First_k(a) = \{a\}, a \in T$(**basis**).

$\oplus_k: 2^{T^*} \times 2^{T^*} \rightarrow 2^{T^{\leq k}}$. $A \oplus_k B = k{:}AB = \{k{:}xy \in T^{\leq k} \mid x \in A, y \in B\}$

Compute $\forall X \in T \cup N$: $First_k(X)$.

for $a \in T$ ***do*** $First_k(a) = \{a\}$ ***od; for*** $A \in N$ ***do*** $First_k(A) := \emptyset$ ***od;***

repeat

for $A \in N$ ***do***

for $A \rightarrow X_1X_2...X_n \in P$ ***do***

for *i:=1 to n* ***do*** $First_k(A) := First_k(A) \oplus_k First_k(X_i)$ ***od***

od od

until $First_k(A)$ *does not change*

Example G_{Uexp}: $E \to E + T \mid T$

$T \to T * F \mid F$

$F \to a \mid (E)$

F: $First_1(a) = \{a\}$ $First_1((E)) = \{(\}$

$\therefore (a, F) \to^{F\to a} (a, a)$ $((, F) \to^{F\to(E)} ((, (E))$ ***deterministic!***

T: $First_1(T * F) = \{a, (\}$ $First_1(F) = \{a, (\}$

$\therefore (a, T) \to^{T\to T*F} (a, T * F)$ $(a, T) \to^{T\to F} (a, F)$ ***non-deterministic!***

$((, T) \to^{T\to T*F} ((, T * F)$ $((, T) \to^{T\to F} ((, F)$ ***non-deterministic!***

E: $First_1(E + T) = \{a, (\}$ $First_1(T) = \{a, (\}$

$\therefore (a, E) \to^{E\to E*T} (a, E + T)$ $(a, E) \to^{E\to T} (a, T)$ ***non-deterministic!***

$((, E) \to^{E\to E*T} ((, E + T)$ $((, E) \to^{E\to T} ((, T)$ ***non-deterministic!***

$\therefore$ *Non-deterministic for* ***T*** *and* ***E***!

A grammar rule $A \to A\alpha \in P$ *is said to be* ***left recursive***.

If $A \to A\alpha \in P$, $A \Rightarrow^* A\alpha^*$ *does* ***not terminate***! $\therefore\ {}^{\exists}A \to \beta \in P$

$A \to A\alpha \mid \beta$, *then* $A \Rightarrow^* \beta\alpha^*$.

$First_k(A\alpha) \cap First_k(\beta) \neq \emptyset$, *since* $First_k(A\alpha) \supseteq First_k(A) \supseteq First_k(\beta)$.

$\therefore$ *If a grammar has* ***left recursive*** *rule, the* ***left parser*** *is* ***non-deterministic***!

Change the ***left recursion*** *to* ***right recursion***.

$A \to A\alpha \mid \beta \Rightarrow \quad A \to \beta A', A' \to \alpha A' \mid \varepsilon$. $(A \Rightarrow \beta A' \Rightarrow^* \beta\alpha^*)$.

G_{Dexp}': $E \to T E'$ $\qquad (E \to T + E \mid T)$

$E' \to + T E' \mid \varepsilon$

$T \to F T'$ $\qquad (T \to F * T \mid F)$

$T' \to * F T' \mid \varepsilon$

$F \to a \mid (E)$

Left factoring $A \to \alpha\beta \mid \alpha\gamma \Rightarrow \quad A \to \alpha A', A' \to \beta \mid \gamma$.

F: $First_1(a) = \{a\}$, $First_1((E)) = \{(\}$

$(a, F) \rightarrow^{F \rightarrow a} (a, a)$

$((, F) \rightarrow^{F \rightarrow (E)} ((, (E))$ **deter.!**

T': $First_1(*FT') = \{*\}$, $First_1(\varepsilon) = \{\varepsilon\}$

$(*, T') \rightarrow^{T \rightarrow FT'} (a, *FT')$

$(\varepsilon, T') \rightarrow^{T' \rightarrow \varepsilon} (\varepsilon, \varepsilon)$ **non-deter.!**

T: $First_1(FT') = \{a, (\}$ unique rule **deter.!**

$(a, T) \rightarrow^{T \rightarrow FT'} (a, FT')$,

$((, T) \rightarrow^{T \rightarrow FT'} ((, FT')$

E': $First_1(+TE') = \{+\}$ $First_1(\varepsilon) = \{\varepsilon\}$

$(+, E') \rightarrow^{E' \rightarrow +TE'} (+, +TE')$

$(\varepsilon, E') \rightarrow^{E' \rightarrow \varepsilon} (\varepsilon, \varepsilon)$ **non-deter.!**

E: $First_1(TE') = \{a, (\}$ unique rule **deter.!**

$(a, E) \rightarrow^{E \rightarrow TE} (a, TE')$,

$((, E) \rightarrow^{E \rightarrow TE} ((, TE')$

*We are **happy** except for $E' \rightarrow \varepsilon$ and $T' \rightarrow \varepsilon$.*
What can I do?

Consider $Follow_k(E')$ *and* $Follow_k(T')$ *or k:z in the figure!*

$Follow_k: N \rightarrow 2^{T^{\leq k}}$.

$Follow_k(A) = \{k{:}z \in T^{\leq k} \mid S \Rightarrow^* \alpha A z,\ z \in T^*\}$ ***finite***.

$Follow_k(T') = First_k(E') \cup Follow_k(E) = \{+,), \varepsilon\}$, $Follow_k(E') = \{), \varepsilon\}$

T': $First_1(*\ F\ T') = \{*\}$

$First_1(\varepsilon) \oplus_1 Follow_1(T') = \{+,), \varepsilon\}$ *two sets are* ***disjoint***!

$(*, T') \rightarrow^{T' \rightarrow *FT'} (*, *\ F\ T')$

$(+, T') \rightarrow^{T' \rightarrow \varepsilon} (+, \varepsilon)$, $(), T') \rightarrow^{T' \rightarrow \varepsilon} (), \varepsilon)$, $(\varepsilon, T') \rightarrow^{T' \rightarrow \varepsilon} (\varepsilon, \varepsilon)$

deterministic!

E': $First_1(+\ T\ E') = \{+\}$

$First_1(\varepsilon) \oplus_1 Follow_1(E') = \{), \varepsilon\}$ *two sets are* ***disjoint***!

$(+, E') \rightarrow^{E' \rightarrow +TE'} (+, +\ T\ E')$

$(), E') \rightarrow^{E' \rightarrow \varepsilon} (), \varepsilon,)$, $(\varepsilon, E') \rightarrow^{E' \rightarrow \varepsilon} (\varepsilon, \varepsilon)$ ***deterministic!***

Parsing table for G_{Dexp}.: $SLL_1PT[\mathbf{N}, T] \to 2^P$.

$A \to \alpha$	$First_1(\alpha) \oplus_1 Follow_1(A)$	
G_{Dexp}: $E \to T\,E'$	$\{a, (\}$	
$E' \to +\,T\,E' \mid \varepsilon$	$\{+\}$	$\{), \varepsilon\}$
$T \to F\,T'$	$\{a, (\}$	
$T' \to *\,F\,T' \mid \varepsilon$	$\{*\}$	$\{+,), \varepsilon\}$
$F \to a \mid (\,E\,)$	$\{a\}$	$\{(\}$

	a	(	*	+	)	ε(\$)
E	$E \to T\,E'$	$E \to T\,E'$				
E'				$E' \to +\,T\,E'$	$E' \to \varepsilon$	$E' \to \varepsilon$
T	$T \to F\,T'$	$T \to F\,T'$				
T'			$T' \to *\,F\,T'$	$T' \to \varepsilon$	$T' \to \varepsilon$	$T' \to \varepsilon$
F	$F \to a$	$F \to (\,E\,)$				

Parsing table for G_{Dexp}.: $SLL_1PT[\mathbf{N}, T] \rightharpoonup P$. $\therefore$ ***deterministic***!

Ex.	*a*	*(*	*	+	*)*	ε($)
E	$E \to T\,E'$	$E \to T\,E'$				
E'				$E' \to +\,T\,E'$	$E' \to \varepsilon$	$E' \to \varepsilon$
T	$T \to F\,T'$	$T \to F\,T'$				
T'			$T' \to *\,F\,T'$	$T' \to \varepsilon$	$T' \to \varepsilon$	$T' \to \varepsilon$
F	$F \to a$	$F \to (\,E\,)$				

$(a * a + a, \mathbf{E}) \Rightarrow_L^{(a,E)\to(a,TE')} (a * a + a, \mathbf{T}\,E') \Rightarrow_L^{(a,T)\to(a,FT')} (a * a + a, \mathbf{F}\,T'\,E')$

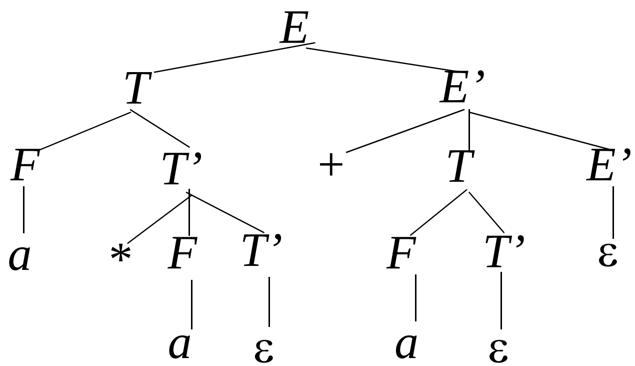
$\Rightarrow_L^{(a,F)\to(a,a)} (a * a + a, \mathbf{a}\,T'\,E') \Rightarrow_L^{(a,a)\to(\varepsilon,\varepsilon)} (* a + a, \mathbf{T'}\,E')$

$\Rightarrow_L^{(*,T')\to(*,*FT')} (* a + a, \mathbf{*}\,F\,T'\,E') \Rightarrow_L^{(*,*)\to(\varepsilon,\varepsilon)} (a + a, \mathbf{F}\,T'\,E') \Rightarrow_L^{(a,F)\to(a,a)} (a + a, \mathbf{a}\,T'\,E')$

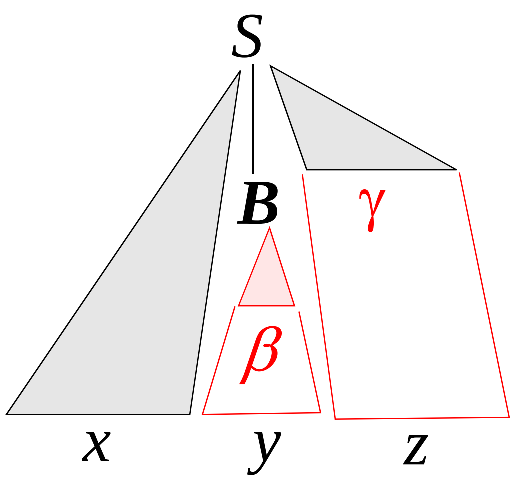
$\Rightarrow_L^{(a,a)\to(\varepsilon,\varepsilon)} (+ a, \mathbf{T'}\,E') \Rightarrow_L^{(+,T')\to(+,\varepsilon)} (+ a, \mathbf{E'}) \Rightarrow_L^{(+,E')\to(+,+TE')} (+ a, \mathbf{+}\,T'\,E')$

$\Rightarrow_L^{(+,+)\to(\varepsilon,\varepsilon)} (a, \mathbf{T'}\,E') \Rightarrow_L^{(a,T')\to(a,FT')} (a, \mathbf{F}\,T'\,E') \Rightarrow_L^{(a,F)\to(a,a)} (a, \mathbf{a}\,T'\,E')$

$\Rightarrow_L^{(a,a)\to(\varepsilon,\varepsilon)} (\varepsilon, \mathbf{T'}\,E') \Rightarrow_L^{(\varepsilon,T')\to(\varepsilon,\varepsilon)} (\varepsilon, \mathbf{E'}) \Rightarrow_L^{(\varepsilon,E')\to(\varepsilon,\varepsilon)} (\varepsilon, \varepsilon).$



Adding lookahead string k:yz $\in T^{\leq k}$ *for* ***guess B as*** β.



$(x, B) \rightarrow_L (x, \beta) \in (T^{\leq k} \times V^*) \times (T^{\leq k} \times V^*)$ *for* $B \rightarrow \beta \in P$ *where*

$x \in First_k(\beta) \oplus_k Follow(B) = k{:}yz.$

LL(k) Parser

Left-to-right Scan in Leftmost derivation with k-lookahead symbols

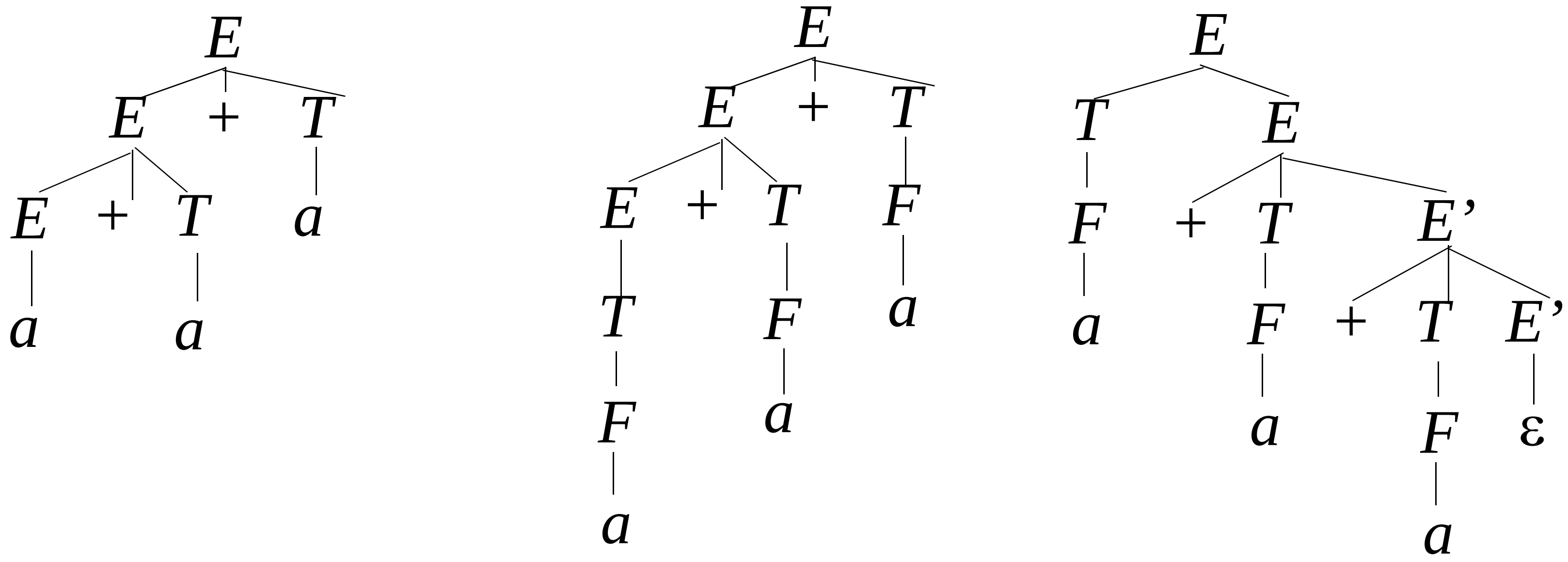
Strong LL(k) Parser(SLL(k) Parser)

SLL(k) grammars $\subset$ *LL(k) grammars*

But SLL(1) grammars = LL(1) grammars

*Compare three grammars for a + a + a or a * a * a*

G_{exp}: $E \to E + T \mid T * F \mid a \mid (E)$	$E \to E + T \mid T$	$E \to T\,E'$
		$E' \to +\,T\,E' \mid \varepsilon$
$T \to T * F \mid a \mid (E)$	$T \to T * F \mid F$	$T \to F\,T'$
		$T' \to *\,F\,T' \mid \varepsilon$
$F \to a \mid (E)$	$F \to a \mid (E)$	$F \to a \mid (E)$



good! *unit production* *+ is right associative*

How about right parsers?