6.A.1 Rewriting systems for PDA's and FA's

*We have defined a **rewriting system** for a FA*

$F = (Q, T, \delta_F, q_0, F)$ *with* $\delta_F: Q \times T^* \rightarrow 2^Q$ *as*

$R_F = (Q \times T^*, \rightarrow_F, (q_0, x), \{(f, \varepsilon) \mid x \in T^*, f \in F\}$ *where*

$\rightarrow_F: = \{(q, x) \rightarrow_F (p, \varepsilon) \in (Q \times T^*) \times (Q \times T^*) \mid p \in \delta_F(q, x)\}.$

Note that $\rightarrow_F \subseteq (Q \times T^*) \times (Q \times T^*)$ *is a binary operation on*

***configurations**(ID's)* $Q \times T^*$: ***configuration rewriting system**.*

*Compare **configuration rewritings** with **state transition functions*** δ_F.

$(q_0, x_1x_2 \ldots x_n) \Rightarrow_F^{(q_0, x_1) \rightarrow (q_1, \varepsilon)} (q_1, x_2 \ldots x_n) \Rightarrow_F \ldots \Rightarrow_F$

$(q_{n-1}, x_n) \Rightarrow_F^{(q_{n-1}, x_n) \rightarrow (q_n, \varepsilon)} (q_n, \varepsilon)$, *if and only if,*

$q_n \in \delta(\ldots\delta(\delta(q_0, x_1x_2\ldots x_n))\ldots) =^{q_1 \in \delta_F(q_0, x_1)} \delta(\ldots\delta(q_1, x_2\ldots x_n))\ldots) = \ldots$

$= \delta(q_{n-1}, x_n) =^{q_n \in \delta_F(q_{n-1}, x_n)} q_n.$

*We define a **rewriting system** for a PDA*

$P = (Q, T, \Gamma, \delta_P, q_0, Z_0, F)$ with $\delta_P: Q \times T^* \times \Gamma^* \rightarrow 2^{Q \times T^* \times \Gamma^*}$ as

$R_P = (Q \times T^* \times \Gamma^*, \rightarrow_P, (q_0, x, Z_0), \{(f, \varepsilon, \alpha) | x \in T^*, f \in F, \alpha \in \Gamma^*\}$

$\rightarrow_P: = \{(q, x, \alpha) \rightarrow_F (p, \varepsilon, \beta) \in (Q \times T^* \times \Gamma^*) \times (Q \times T^* \times \Gamma^*)$
$| \ (p, \beta) \in \delta_P(q, x, \alpha)\}.$

Note that $\rightarrow_P \subseteq (Q \times T^* \times \Gamma^*) \times (Q \times T^* \times \Gamma^*)$ *is a binary operation on*

***configurations**(ID's)* $Q \times T^* \times \Gamma^*$*: **configuration rewriting system**.*

*Compare **configuration rewritings** with **state transition functions*** δ_P.

$(q_0, x_1x_2 \ldots x_n, \alpha_1\gamma) \Rightarrow_P^{(q_0, x_1, \alpha_1)\rightarrow(q_1, \varepsilon, \alpha_2)} (q_1, x_2 \ldots x_n, \alpha_2\gamma) \Rightarrow_P \ldots \Rightarrow_P$

$(q_{n-1}, x_n, \alpha_{n-1}\gamma') \Rightarrow_P^{(q_{n-1}, x_n, \alpha_{n-1})\rightarrow(q_n, \varepsilon, \alpha_n)} (q_n, \varepsilon, \alpha_n\gamma')$, *if and only if,*

$(q_0, x_1x_2 \ldots x_n, \alpha_1\gamma) \vdash_P^{(q_1,\alpha_2)\in\delta_P(q_0,x_1,\alpha_1)} (q_1, x_2 \ldots x_n, \alpha_2\gamma) \vdash_P \ldots \vdash_P$

$(q_{n-1}, x_n, \alpha_{n-1}\gamma') \vdash_P^{(q_n,\alpha_n)\in\delta_P(q_{n-1},x_n,\alpha_{n-1})} (q_n, \varepsilon, \alpha_n\gamma')$.

We ***extend*** $\rightarrow_P$ *of a* ***rewriting system*** $R_P = (Q \times T^* \times \Gamma^*, \rightarrow_P, \iota, \Phi)$*, by* ***adding*** *lookahead input string(*$y \in T^*$*) and lookback stack string(*$\gamma \in \Gamma^*$*).*

$\rightarrow_P = \{(q, xy, \alpha\gamma) \rightarrow_P (p, y, \beta\gamma) \in (Q \times T^* \times \Gamma^*) \times (Q \times T^* \times \Gamma^*)$

$\quad | \; p, q \in Q, x, y \in T^*, \alpha, \beta, \gamma \in \Gamma^*\} \subseteq (Q \times T^* \times \Gamma^*) \times (Q \times T^* \times \Gamma^*).$

(3) $\iota = (q_0, x, \gamma_0) \in Q \times T^* \times \Gamma^*$*, and*

(4) $\Phi = \{(p, \varepsilon, \gamma) | \; p \in Q, \gamma \in \Gamma^*\}$

Note that the 2nd ***Conf.*** T^* *is* ***read only*** *input(*$xy \rightarrow y$*) string and the 3rd* ***Conf.*** Γ^* *is a* ***read/write stack***$(\alpha\gamma \rightarrow \beta\gamma)$ *string.*

6.A.2 Left Parsers for Context-free Grammars

Let G = (N, T, P, S) be a ***context-free grammar****. Then we* ***define***

A ***PDA*** $R_L = (T^* \times (N \cup T)^*, \rightarrow_L, \iota = (x, S), \Phi = \{(\varepsilon, \varepsilon)\})$ *as*

a ***Left (gues-verify) parser*** *for the* ***context-free*** *grammar G where*

$\rightarrow_L = \{(\varepsilon, A) \rightarrow_L (\varepsilon, \alpha) \mid A \rightarrow \alpha \in P\}$ ***guess A as*** α *onto* ***stack***

$\cup \{(a, a) \rightarrow_L (\varepsilon, \varepsilon) \mid a \in \Sigma\}$. ***verify*** $a \in \Sigma$ *with input and* ***stack***

$L(R_L) = \{x \in \Sigma^* \mid (x, S) \Rightarrow_L^* (\varepsilon, \varepsilon) \in \Phi\}$.

We ***add*** *P and* ***output function*** $\tau_L: \rightarrow_L^* \rightarrow P^*$ *to the* ***left parser*** *as*

$P_L = (T^* \times (N \cup T)^*, \rightarrow_L, (x, S), \{(\varepsilon, \varepsilon)\}, P, \tau_L)$ *with*

$\tau_L((\varepsilon, A) \rightarrow_L (\varepsilon, \alpha)) = A \rightarrow \alpha \in P$ *and*

$\tau_L((a, a) \rightarrow_L (\varepsilon, \varepsilon)) = \varepsilon$. *Then*

$(x, S) \Rightarrow_L^{\theta_L} (\varepsilon, \varepsilon)$, *iff* $S \Rightarrow_{lm}^{\pi_L} x$ *where* $\tau_L(\theta_L) = \pi_L \in P^*$ ***left parse*** *of x,*

$|\theta_L| = |\pi_L| + |x|$ $|\pi_L|$-***guess*** *actons and* $|x|$-***verify*** *actions.*

*Consider a **leftmost derivations** in G and **rewritings** in the **left parser*** P_L.

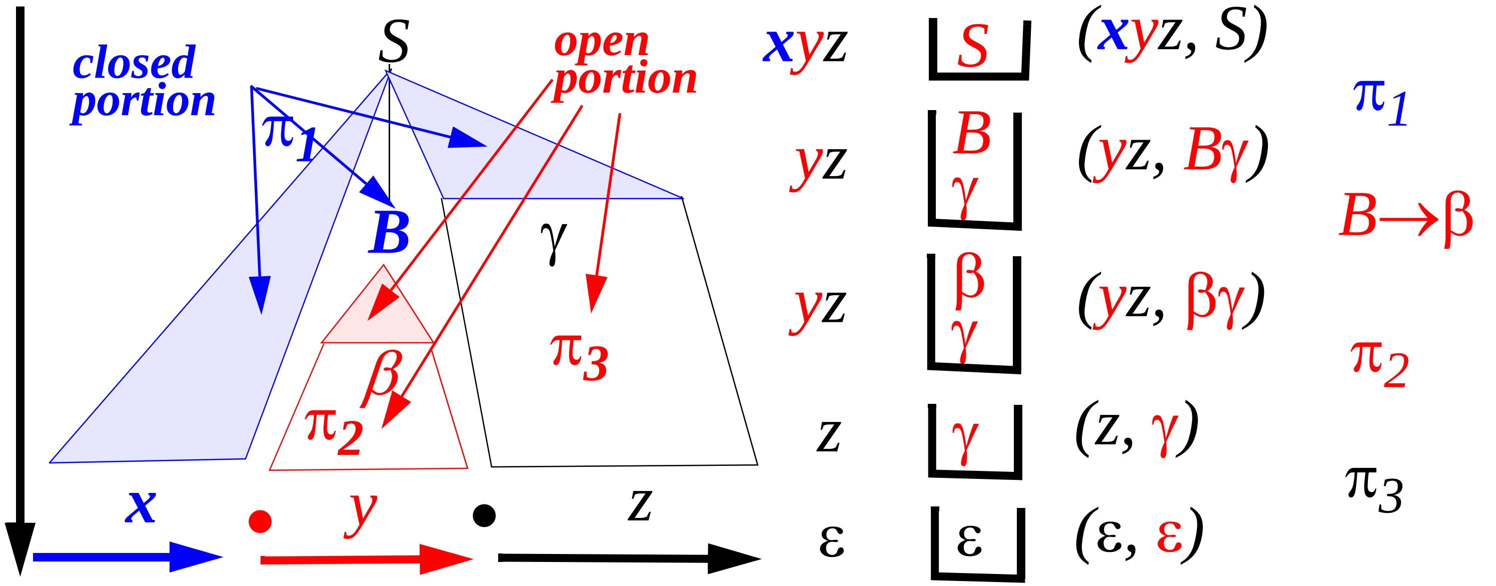
$$S \Rightarrow_{lm}^{\pi_1} xB\gamma \Rightarrow_{lm}^{B\to\beta} x\beta\gamma \Rightarrow_{lm}^{\pi_2} xy\gamma \Rightarrow_{lm}^{\pi_3} xyz \text{ and}$$

$$(xyz, S) \Rightarrow_L^{\theta_1} (yz, B\gamma) \Rightarrow_L^{B\to\beta} (yz, \beta\gamma) \Rightarrow_L^{\theta_2} (z, \gamma) \Rightarrow_L^{\theta_3} (\varepsilon, \varepsilon).$$

$|\theta_1| = |\pi_1|+|x|$, $|\theta_2| = |\pi_2|+|y|$, $|\theta_3| = |\pi_3|+|z|$. $\tau(\theta_1)=\pi_1$, $\tau(\theta_2)=\pi_2$, $\tau(\theta_3)=\pi_3$.

$$\tau(\theta) = \tau(\theta_1 \cdot B\to\beta \cdot \theta_2\theta_3) = \pi_1 \cdot B\to\beta \cdot \pi_2\pi_3 = \pi.$$

$|\pi_1 \cdot B\to\beta \cdot \pi_2\pi_3|$ ***guess*** *actions* + $|xyz|$ ***verify*** *actions.*|



Example G_{Uexp}: $E \rightarrow E + T \mid T * F \mid a \mid (E)$

$T \rightarrow T * F \mid a \mid (E)$

$F \rightarrow a \mid (E)$

$(a+a*a, E)$

$\Rightarrow^{E\rightarrow E+T}(a+a*a, E+T)$ or $\Rightarrow^{E\rightarrow T*F}(a+a*a, T*F)$ or $\Rightarrow^{E\rightarrow (E)}(a+a*a, (E))$ or $\Rightarrow^{E\rightarrow a}(a+a*a, a)$

$\Rightarrow^{E\rightarrow a} (a+a*a, a+T)$ $\quad\Rightarrow^{T\rightarrow T*F}(a+a*a, T*F*F) \Rightarrow_L^{a\neq(}$ Error! $\quad\Rightarrow^{a} (+a*a, \varepsilon)$

$\Rightarrow^{a} (+a*a, +T)$ $\quad\Rightarrow^{T\rightarrow a}(a+a*a, a*F*F)$ or $\Rightarrow^{T\rightarrow (E)}(a+a*a, (E)*F*F) \Rightarrow^{+\neq\varepsilon}$ Error!

$\Rightarrow^{+} (a*a, T)$ $\quad\Rightarrow^{a}(+a*a, *F*F)$ $\quad\Rightarrow_L^{a\neq(}$ Error!

$\Rightarrow^{T\rightarrow T*F} (a*a, T*F)$ $\quad\Rightarrow^{+\neq *}$ Error!

$\Rightarrow^{T\rightarrow a} (a*a, a*F)$

$\Rightarrow^{a} (*a, *F)$

$\Rightarrow^{*} (a, F)$

$\Rightarrow^{F\rightarrow a} (a, a)$

$\Rightarrow^{a} (\varepsilon, \varepsilon)$

Lemma 6.A.1 *If* $(xy, \gamma) \Rightarrow_L^{\theta_L} (y, \delta)$ *in* P_L, *then* $\gamma \Rightarrow_{lm}^{\tau_L(\theta_L)} x\delta$ *in G*

and $|\theta_L| = |\tau_L(\theta_L)| + |x|$.

Proof Induction on $|\theta_L| \in \mathbb{N}$.

i) $|\theta_L| = 0$: $\theta_L = \varepsilon$. $(xy, \gamma) = (y, \delta)$. $\therefore$ $x = \varepsilon$, $\gamma = \delta$, *and* $\tau_L(\theta_L) = \varepsilon$.

ii) $\theta_L = a \cdot \theta_L' \in \rightarrow_L^{+}$. *where* $a \in \rightarrow_L$ *and* $\theta_L' \in \rightarrow_L^{*}$.

ii.1) $a = (\varepsilon, A) \rightarrow_L (\varepsilon, \alpha)$ ***guess*** $A \rightarrow \alpha \in P$ *action.*

$(xy, \gamma) = (xy, A\gamma') \rightarrow_L^{A\rightarrow\alpha} (xy, \alpha\gamma') \Rightarrow_L^{\theta_L'} (y, \delta)$ *in* P_L.

$\therefore$ $\alpha\gamma' \Rightarrow_{lm}^{\pi_L'} x\delta$ *in G and* $|\theta_L| = |\tau_L(\theta_L')| + |x|$ *by IH.*

$\gamma = A\gamma' \Rightarrow_{lm}^{A\rightarrow\alpha} \alpha\gamma' \Rightarrow_{lm}^{\tau_L(\theta_L')} x\delta$ *in G and*

$|\theta_L| = 1 + |\tau_L(\theta_L')| + |x| = |\tau_L(\theta_L)| + |x|$.

ii.2) $a = (a, a) \rightarrow_L (\varepsilon, \varepsilon)$ ***verify*** $a \in T$ *action*

$(xy, \gamma) = (ax'y, a\gamma') \rightarrow_L^{a} (x'y, \gamma') \Rightarrow_L^{\theta_L'} (y, \delta)$ *in* P_L.

$\therefore \gamma' \Rightarrow_{lm}^{\pi_L'} x'\delta$ *in G and* $|\theta_L| = |\tau_L(\theta_L')| + |x'|$ *by IH.*

$\gamma = a\gamma' \Rightarrow_{lm}^{\tau_L(\theta_L')} ax'\delta = x\delta$ *in G and*

$|\theta_L| = 1 + |\tau_L(\theta_L')| + |x'| = |\tau_L(\theta_L)| + |x|$

Apply $y = \varepsilon$, $\gamma = S$, *and* $\delta = \varepsilon$ *in* ***Lemma A***.

If $(x, S) \Rightarrow_L^{\theta_L} (\varepsilon, \varepsilon)$ *in* P_L, *then* $S \Rightarrow_{lm}^{\tau_L(\theta_L)} x$ *in G*

and $|\theta_L| = |\tau_L(\theta_L)| + |x|$.

$\therefore L(P_L) \subseteq L(G)$, $\tau_L(\theta_L) = \pi_L$, *and* $|\pi_L| = |\theta_L| - |x|$.

Lemma 6.A.2 *If* $\gamma \Rightarrow_{lm}^{\pi_L} x\delta$ *in G,* $\delta = \varepsilon$ *or* $1{:}\delta \in N$, *then*

$(xy, \gamma) \Rightarrow_L^{\theta_L} (y, \delta)$ *in* P_L, $\tau_L(\theta_L) = \pi_L$, *and* $|\theta_L| = |\pi_L| + |x|$.

Proof Induction on $|\pi_L| \in \mathbb{N}$.

i) $\pi_L = \varepsilon$: $x = \varepsilon$, $\gamma = \delta$, *and* $(y, \gamma) \Rightarrow_L^{\varepsilon} (y, \gamma)$. $\therefore \tau_L(\varepsilon) = \pi_L = \varepsilon$.

ii) $\pi_L = \pi_L' \cdot \theta_L'$, $\pi_L' \in P^*$, $\exists \theta_L \in \rightarrow_L^*$.∋. $\tau_L(\theta_L) = \pi_L$.

$$\gamma \Rightarrow_{lm}^{\pi_L'} x'\delta' = x'A\delta'' \Rightarrow_{lm}^{A\to\alpha} x'\alpha\delta'' = x'z\delta = x\delta, \text{ where}$$

$$|\theta_L'| = |\pi_L'| + |x'|,\ \delta' = A\delta'',\ \alpha\delta'' = z\delta,\ x'z = x.$$

$$\therefore (xy, \gamma) = (x'zy, \gamma) \Rightarrow_L^{\theta_L'} (zy, \delta') = (zy, A\delta'')$$

$$\Rightarrow_L^{A\to\alpha} (zy, \alpha\delta'') = (zy, z\delta) \Rightarrow_L^{|z|} (y, \delta).$$

$$\therefore |\theta_L| = |\theta_L'| + 1 + |z| = |\pi_L'| + |x'| + 1 + |z| = |\pi_L| + |x|.$$

$$\tau_L(\theta_L) = \pi_L'\cdot A \to \alpha\cdot\varepsilon = \pi_L.$$

Apply $y = \varepsilon$, $\gamma = S$, *and* $\delta = \varepsilon$ *in* ***Lemma B***.

If $S \Rightarrow_{lm}^{\pi_L} x$ *in G, then* $(x, S) \Rightarrow_L^{\theta_L} (\varepsilon, \varepsilon)$ *in* P_L,

and $\tau_L(\theta_L) = \pi_L$, $|\theta_L| = |\pi_L| + |x|$.

$\therefore L(G) \subseteq L(P_L)$, $^\forall\pi_L$: ***left parse*** *of* x $\tau_L(\theta_L) = \pi_L$, *and* $|\theta_L| = |\pi_L| + |x|$.

Theorem 6.A

$L(G) = L(P_L)$, $^\forall x \in L(G)$ *in G*, $\tau_L(\theta_L) = \pi_L$ *in* P_L, *and* $|\theta_L| = |\pi_L| + |x|$.

6.A.3 Right Parsers for Context-free Grammars

A **PDA** $P_R = (T^* \times (N \cup T)^*, \rightarrow_R, (x, \varepsilon), \{(\varepsilon, S)\})$ is

a **Right (shift-reduce) parser** for the **context-free** grammar G where

$\rightarrow_R = \{(a, \varepsilon) \rightarrow_R (\varepsilon, a) | a \in \Sigma\}$ **shift**(push) input $a \in \Sigma$ onto **stack**

$\cup \{(\varepsilon, \alpha^R) \rightarrow_R (\varepsilon, A) | A \rightarrow \alpha \in P\}$. **reduce** α^R **to A** onto **stack**

$L(P_R) = \{x \in \Sigma^* | (x, \varepsilon) \Rightarrow_R^* (\varepsilon, S) \in \Phi\}$.

We **add** P an **output function** $\tau_R: \rightarrow_R^* \rightarrow P^*$ to the **right parser** as

$P_R = ((T^* \times \Gamma^*), \rightarrow_R, (x, \varepsilon), \{(\varepsilon, S)\}, P, \tau_R)$ with

$\tau_R((a, \varepsilon) \rightarrow_R (\varepsilon, a)) = \varepsilon$ and

$\tau_R((\varepsilon, \alpha^R) \rightarrow_R (A, \varepsilon)) = A \rightarrow \alpha \in P$. Then

$(x, \varepsilon) \Rightarrow_R^{\theta_R} (\varepsilon, S)$, iff $S \Rightarrow_{rm}^{\pi_R^R} x$, $\tau_R(\theta_R) = \pi_R^R \in P^*$ **right parse** of x and

$|\theta_R| = |\pi_R^R| + |x|$ $|x|$-**shift** actons and $|\pi_R^R|$-**reduce** actions.

Example G_{Uexp}: $E \to E + T \mid T * F \mid a \mid (E)$

$$T \to T * F \mid a \mid (E)$$
$$F \to a \mid (E)$$

$(a+a*a, \varepsilon)$

$\Rightarrow^{a} (+a*a, a)$

$\Rightarrow^{E\to a} (+a*a, E)$ or $\Rightarrow^{T\to a} (+a*a, T)$ or $\Rightarrow^{F\to a} (+a*a, F)$

$\Rightarrow^{+} (a*a, +E)$ $\Rightarrow^{+} (a*a, +T)$ $\Rightarrow^{+} (a*a, +F)$

$\Rightarrow^{a} (*a, a+E)$ $\Rightarrow^{a} (*a, a+T)$ $\Rightarrow^{a} (*a, a+F)$

$\Rightarrow^{T\to a} (*a, T+E)$ $\Rightarrow^{F\to a} (*a, F+T)$

$\Rightarrow^{*} (a, *T+E)$ $\Rightarrow^{*} (a, *F+T)$ or $\Rightarrow^{T\to T*F} (*a, T)$ $\Rightarrow^{T\to T*F} (*a, T)$

$\Rightarrow^{a} (\varepsilon, a*T+E)$ $\Rightarrow^{a} (\varepsilon, a*F+T)$ $\Rightarrow^{*} (a, *T)$

$\Rightarrow^{F\to a} (\varepsilon, F*T+E)$ or $\Rightarrow^{T\to a} (\varepsilon, T*T+E)$ or $\Rightarrow^{E\to a} (\varepsilon, E*T+E)$ $\Rightarrow^{a} (\varepsilon, a*T)$

$\Rightarrow^{T\to T*F} (\varepsilon, T+E)$ $\Rightarrow^{?}$ Error ! $\Rightarrow^{?}$ Error ! $\Rightarrow^{F\to a} (\varepsilon, F*T)$

$\Rightarrow^{E\to E+T} (\varepsilon, E)$

Consider a **rightmost derivations** *in G and* **rewritings** *in* **right parser** P_R.

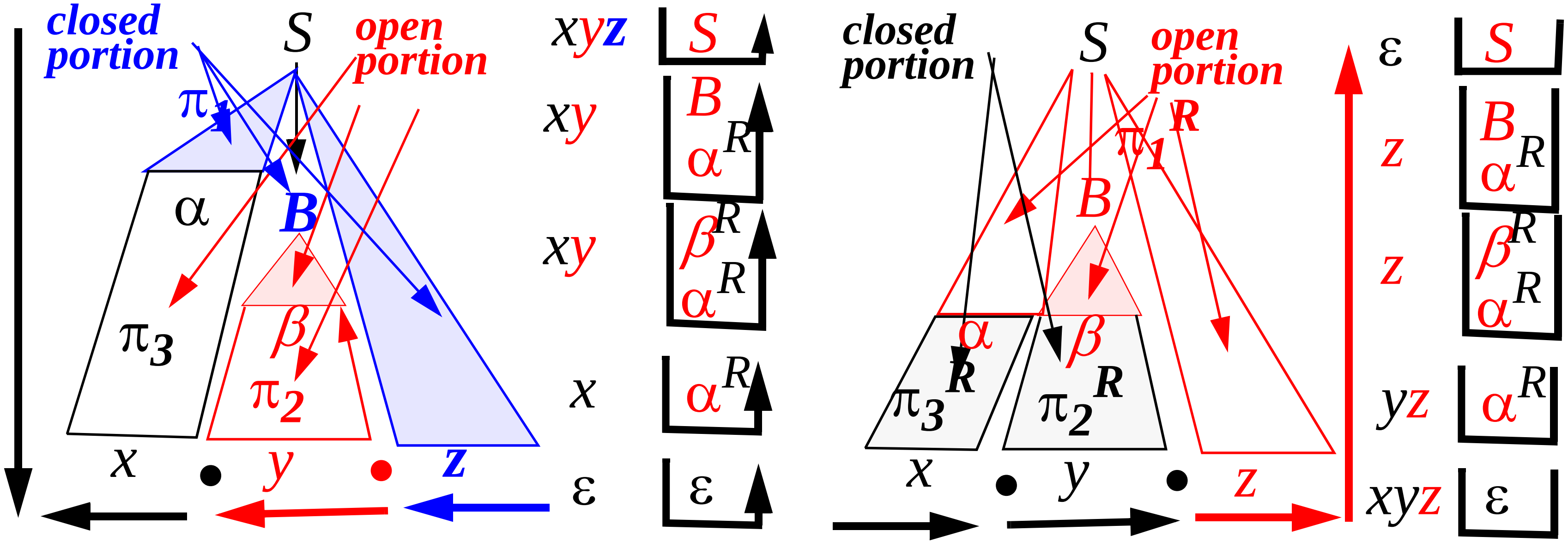
$S \Rightarrow_{rm}^{\pi_1} \alpha B z \Rightarrow_{rm}^{B\to\beta} \alpha\beta z \Rightarrow_{rm}^{\pi_2} xyz \Rightarrow_{rm}^{\pi_3} xyz$

$(xy, S) \Rightarrow_R^{\theta_1} (yz, B\alpha^R) \Rightarrow_R^{B\to\beta} (yz, \beta\alpha^R) \Rightarrow_R^{\theta_2} (x, \alpha^R) \Rightarrow_R^{\theta_3} (\varepsilon, S)$

Rightmost Derivation in Reversed Order

$xyz \Rightarrow_{rm}^{\pi_3^R} \alpha yz \Rightarrow_{rm}^{\pi_2^R} \alpha\beta z \Rightarrow_{rm}^{B\leftarrow\beta} \Rightarrow_{rm} \alpha B z \Rightarrow_{rm} S.$

$(xyz, \varepsilon) \Rightarrow_R^{\theta_3} (yz, \alpha^R) \Rightarrow_R^{\theta_2} (z, \beta^R\alpha^R) \Rightarrow_R^{(\varepsilon,\beta^R)\to(\varepsilon,B)} (z, B\alpha^R) \Rightarrow_R^{\theta_1} (\varepsilon, S)$



Lemma 6.B.1 *If* $(xy, \gamma^R) \Rightarrow_R^{\theta_R} (y, \delta^R)$ *in* P_R, *then* $\delta \Rightarrow_{rm}^{\tau_R(\theta_R)^R} \gamma x$

and $|\theta_R| = |\tau_R(\theta_R)| + |x|$.

Proof Induction *on* $|\theta_R|$

i) $\theta_R = \varepsilon$: $x = \varepsilon$, $\gamma^R = \delta^R$, *and* $\tau_R(\theta_R)^R = \varepsilon$.

ii.1) $\theta_R = A\rightarrow\alpha\cdot\theta_R'$ ***reduce*** $A\rightarrow\alpha \in P$

$(xy, \gamma^R) = (xy, \alpha^R\gamma''^R) \Rightarrow_R^{A\rightarrow\alpha} (xy, A\gamma''^R) = (xy, \gamma'^R) \Rightarrow_R^{\theta_R'} (y, \delta^R)$

and $|\theta_R'| = |\tau_R(\theta_R')| + |x|$.

$\delta^R \Rightarrow_{rm}^{\tau_R(\theta_R')^R} \gamma'^R x = \gamma''^R A x \Rightarrow_{rm}^{A\rightarrow\alpha} \gamma''^R\alpha^R x = \gamma^R x$.

$\therefore |\theta_R| = 1 + |\theta_R'| = 1 + |\tau_R(\theta_R')| + |x| = |\tau_R(\theta_R)| + |x|$.

ii.2) $\theta_R = a\cdot\theta_R'$ ***shift*** $a \in T$

$(xy, \gamma^R) = (ax'y, \gamma^R) \Rightarrow_R^{a} (x'y, a\gamma^R) = (x'y, \gamma) \Rightarrow_R^{\theta_R'} (y, \delta^R)$

and $|\theta_R'| = |\tau_R(\theta_R')| + |x|$.

$\delta^R \Rightarrow_{rm}^{\tau_R(\theta_R')^R} \gamma'^R x' = \gamma^R a x' = \gamma^R x$ *in G and*

$|\theta_R| = 1 + |\theta_R'| = |\tau_R(\theta_R')| + 1 + |x'| = |\tau_R(\theta_R)| + |x|$.

$\therefore L(G) \subseteq L(P_R)$, $^\forall\pi_R$: ***right parse*** *of* x $\tau_R(\theta_R) = \pi_R$, *and* $|\theta_R| = |\pi_R| + |x|$.

Lemma 6.B.2 *If* $\delta \Rightarrow_{rm}^{\pi_R^R} \gamma x$ *in G and* $\gamma = \varepsilon$ *or* $\gamma{:}1 \in N$, *then*

$(xy, \gamma^R) \Rightarrow_R^{\theta_R} (y, \delta^R)$ *in* P_R *and* $|\theta_R| = |\tau_R(\theta_R)| + |x|$.

Proof Induction *on* $|\pi_R|$

i) $\pi_R = \varepsilon$: $\delta = \gamma x$, $(xy, \gamma^R) \Rightarrow_R^x (y, \delta^R)$, $|\theta_R| = |x|$, *and* $\tau_R(\theta_R) = \pi_R = \varepsilon$.

ii) $\pi_R = A\rightarrow\alpha\cdot\pi_R'$, $\pi_R' \neq \varepsilon$: $^\exists\theta_R'$.э. $\tau_R(\theta_R') = \pi_R'$.

$\delta \Rightarrow_{rm}^{\pi_R'^R} \gamma' x' = \delta'' A x' \Rightarrow_{rm}^{A\rightarrow\alpha} \delta''\alpha x' = \gamma x$.

where $|\theta_R'| = |\pi_R'| + |x'|$, $\gamma = \delta''A$, $\delta''\alpha = \gamma z$, $zx' = x$.

$\therefore (xy, \gamma^R) = (zx'y, \gamma^R) \Rightarrow_R^{z} (x'y, \gamma^R) = (x'y, \alpha^R\delta''^R) \Rightarrow_R^{A\to\alpha} (x'y, A\delta''^R) = (x'y, \gamma'^R) \Rightarrow_R^{x'} (y, \delta^R)$.

$\therefore |\theta_R| = |\theta_R'| + 1 + |z'| = |\pi_R'| + |x'| + 1 + |z'| = |\pi_R| + |x|$.

$\therefore L(P_R) \subseteq L(G)$, $\tau_R(\theta_R) = \pi_R$, and $|\pi_R| = |\theta_R| - |x|$.

Theorem 6.B

$L(G) = L(P_R)$, $\forall x \in L(G)$ in G, $\tau_R(\theta_R) = \pi_R$ in P_R, and $|\theta_R| = |\pi_R| + |x|$.