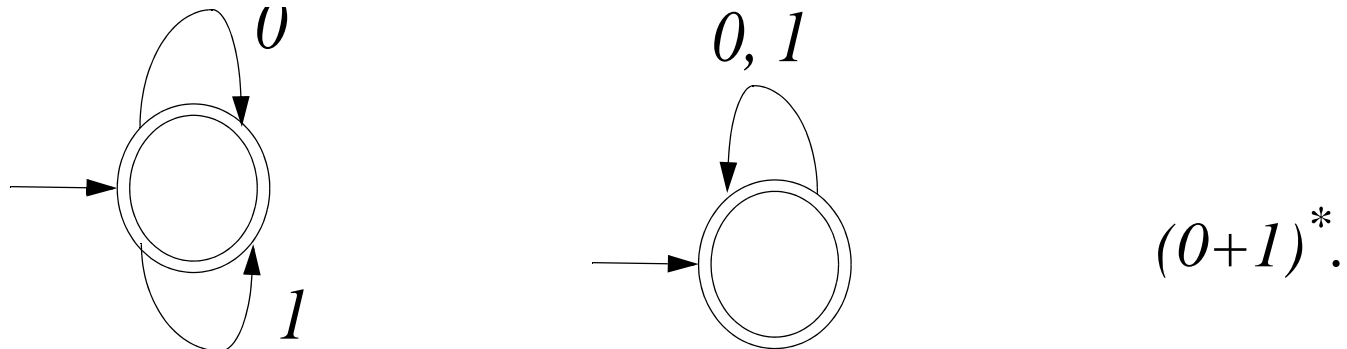
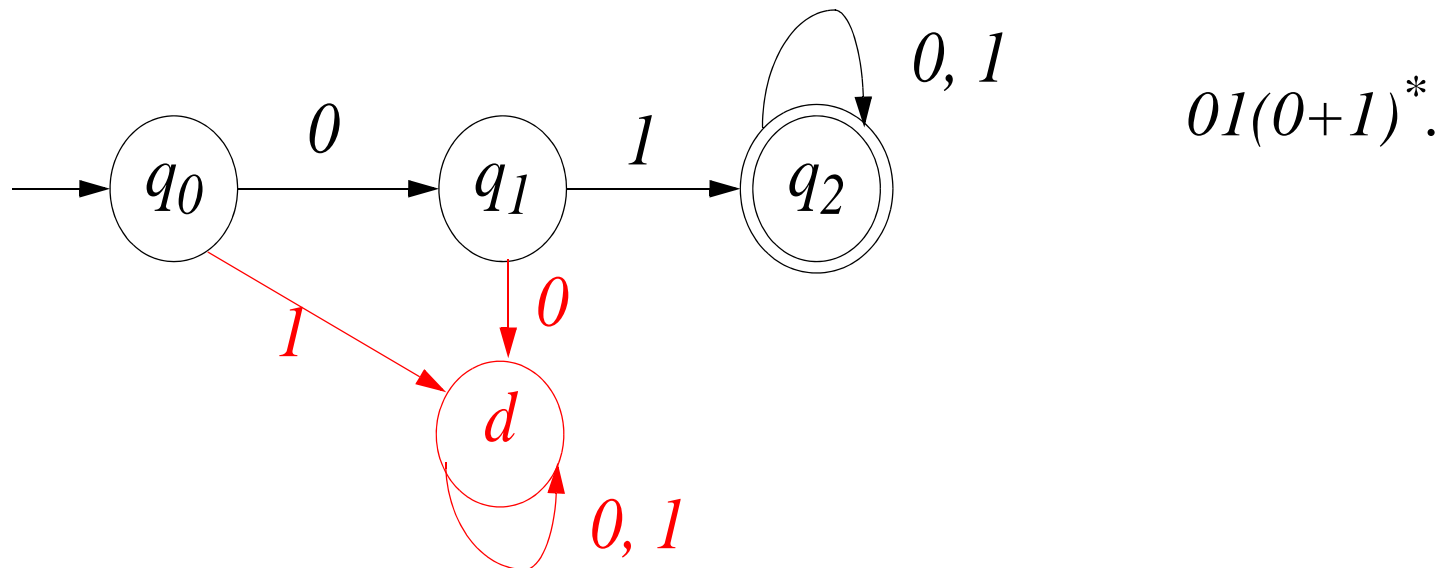


2.B Examples of DFA's Du and Ko's 부교재 Example 2.6-2.15(p28-37).

Example 2.6

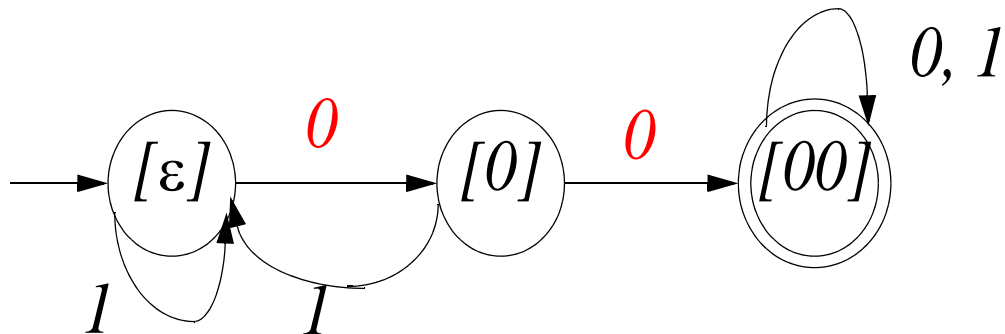


Example 2.7 The set of all binary strings beginning with **prefix 01**.



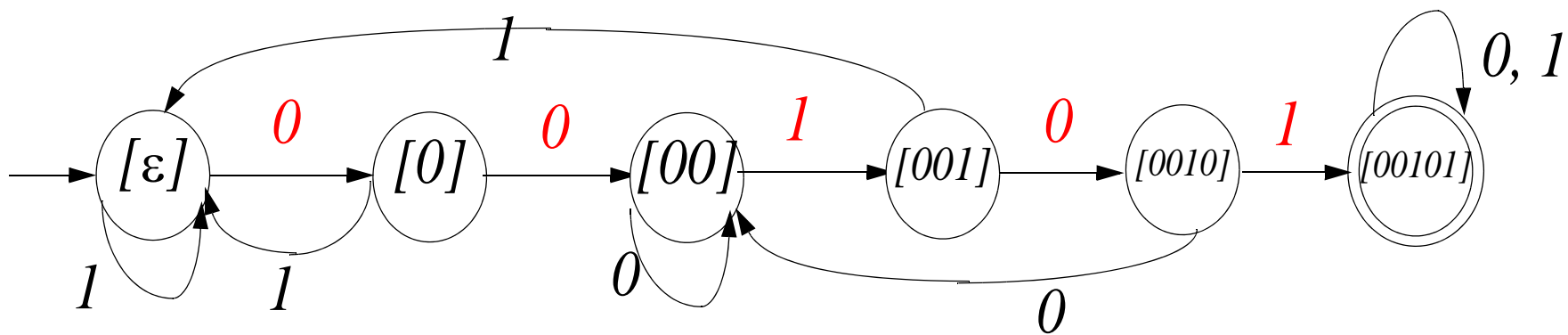
Example 2.8 The set of all binary strings having a **substring 00**.

$$(0+1)^* \mathbf{00} (0+1)^*$$



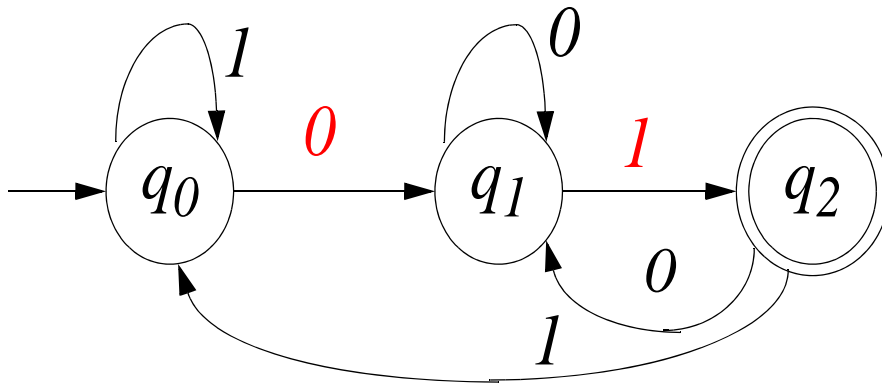
Example 2.9 The set of all binary strings having a **substring 00101**.

$$(0+1)^* \mathbf{00101} (0+1)^*$$



Example 2.10 The set of all binary strings ending with **postfix 01**.

$(0+1)^*01$.



1. Strings **beginning** with $a_1 \dots a_k$. (**prefix** $a_1 \dots a_k$) r.e. $a_1 \dots a_k \Sigma^*$.

$M = (\{q_0, \dots, q_k, d\}, \Sigma, \delta, q_0, \{q_k\})$ ($k+2$) states: q_0, q_1, \dots, q_k and d .

$0 \leq \forall i < k$:

q_i stands for “the **prefix** $a_1 \dots a_i$ of the **prefix** $a_1 \dots a_k$ is found.” ($i \leq k$)

$$\delta(q_i, a_{i+1}) = q_{i+1},$$

$$\forall a \in \Sigma - \{a_{i+1}\}: \delta(q_i, a) = d \quad \text{fail to get prefix } a_1 \dots a_k.$$

d stands for “**fail** to get prefix $a_1 \dots a_k$.”

$$\forall a \in \Sigma, \delta(d, a) = d. \quad \text{dead state}$$

q_k stands for “**prefix** $a_1 \dots a_k$ is already found.” final state

$$\forall a \in \Sigma, \delta(q_k, a) = q_k.$$

2. Strings having a **substring** $a_1 \dots a_k$. (**substring** $a_1 \dots a_k$) r.e $\Sigma^* a_1 \dots a_k \Sigma^*$.

$M = (\{q_0, \dots, q_k\}, \Sigma, \delta, q_0, \{q_k\})$ $(k+1)$ states: q_0, q_1, \dots, q_k

$0 \leq \forall i < k$:

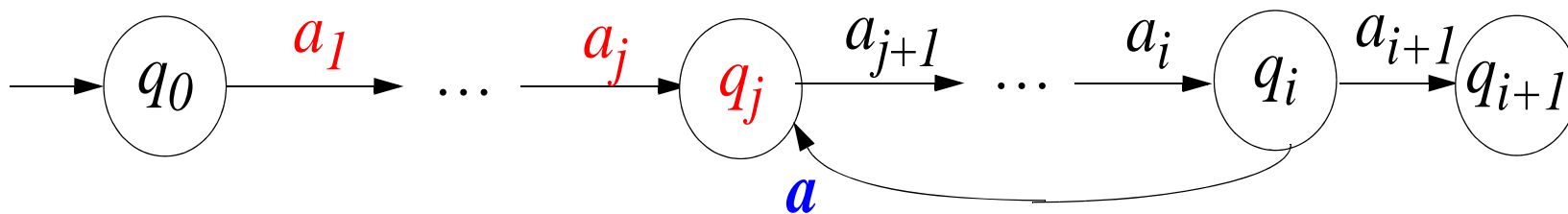
q_i stands for “the **prefix** $a_1 \dots a_i$ of the **substring** $a_1 \dots a_k$ is found.” ($i \leq k$)

$$\delta(q_i, a_{i+1}) = q_{i+1},$$

$$\forall a \in \Sigma - \{a_{i+1}\}: \delta(q_i, a) = q_j.$$

where j ($0 \leq j \leq i$) is the **maximum** index $\exists. a_1 \dots a_j = a_{i-j+2} \dots a_{i-1} a_i a$. ($a_j = a$)

Note that $|a_{i-j+2} \dots a_{i-1} a_i a| = i - (i-j+2) + 1 + 1 = j$.



q_k stands for “**substring** $a_1 \dots a_k$ is already found.” final state

$$\forall a \in \Sigma, \delta(q_k, a) = q_k.$$

3. Strings **ending** with $a_1 \dots a_k$. (**suffix** $a_1 \dots a_k$) r.e. $\Sigma^* a_1 \dots a_k$.

$M = (\{q_0, \dots, q_k\}, \Sigma, \delta, q_0, \{q_k\})$ $(k+1)$ states: q_0, q_1, \dots, q_k

$0 \leq \forall i \leq k$:

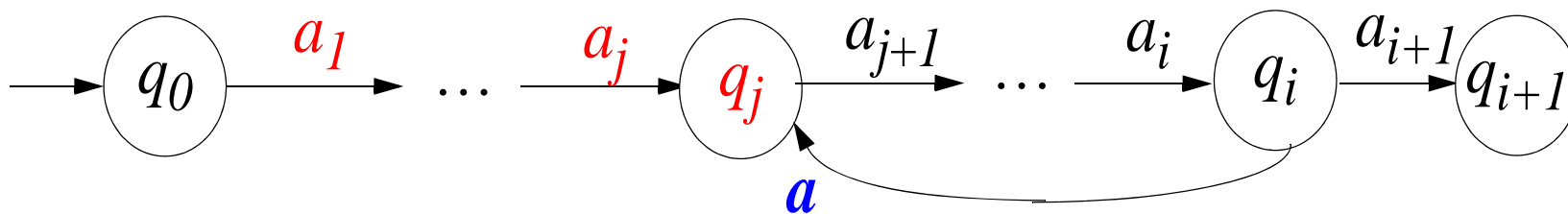
q_i stands for “the **prefix** $a_1 \dots a_i$ of the **suffix** $a_1 \dots a_k$ is found.” ($i \leq k$)

$\delta(q_i, a_{i+1}) = q_{i+1}$;

$\forall a \in \Sigma - \{a_{i+1}\}$: $\delta(q_i, a) = q_j$, if $a \neq a_{i+1}$ (otherwise).

where $j(\leq i)$ is the **maximum** index $\exists. a_1 \dots a_j = a_{i-j+2} \dots a_{i-1} a_i a$.

Note that $|a_{i-j+2} \dots a_{i-1} a_i a| = j$.



In the final state q_k ,

a suffix $a_1 \dots a_k$ is found but search for the next **suffix**, if any,

Example 2.11 *The set of binary expressions of positive integer which are congruent to m modulo k ($0 \leq m < k$).*

Consider k -states q_0, \dots, q_{k-1} ,

q_i stands for “ $y \equiv i \pmod{k}$ ”, $0 \leq i < k$.

basis: $\delta(q_0, \varepsilon) = q_0$ and $\varepsilon \equiv 0 \pmod{k}$.

rec: Suppose $\delta(q_0, x) = q_i$ and $\delta(q_0, xa) = \delta(\delta(q_0, x), a) = \delta(q_i, a) = q_j$.
 $j \equiv xa \pmod{k} \equiv 2i + a \pmod{k}$ for $a \in \{0, 1\}$

Final states = $\{q_m\}$

Special case $m=0$ and no leading zeros

new start state q_0' and dead state d ,

$\delta(q_0', 0) = d, \delta(q_0', 1) = q_{2*0+1 \pmod{k}} = q_{1 \pmod{k}} = q_1.$

$\delta(d, 0) = \delta(d, 1) = d.$

If no leading zeros except 0.

$\delta(q_0', 0) = f, \delta(q_0', 1) = q_1, \delta(f, 0) = \delta(f, 1) = d, \delta(d, 0) = \delta(d, 1) = d.$

Example 2.15 *The set of binary strings in which every block of four consecutive symbols contains a substring 01.*

Consider the complement \bar{L} contains a substring 0000, 1000, 1100, 1110 or 1111.