

Homework #8

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1. 다음 언어가 context-free 인지 아닌지를 **CFL pumping lemma**를 이용하여 보이시오. (4pt, 각각 2pt)

a) $L = \{a^n b^n c^i \mid n \leq i \leq 2n\}$

CFL pumping lemma

a) $L = \{a^n b^n c^i \mid n \leq i \leq 2n\}$

1) $\forall n \geq 0$

2) $\exists z = a^n b^n c^n \in L$

3) $\forall u, v, w, x, y: z = uvwx y, vx \neq \epsilon, |uvwx| \leq n$

i) $0 \leq \forall i \leq n: u = a^{n-i}, vwx = a^i b^{n-i}, y = b^i c^n$

ii) $0 \leq \forall i \leq n: u = a^n b^{n-i}, vwx = b^i c^{n-i}, y = c^i$

these two cover all the cases of z

4) $\forall k \neq 1, uv^k wx^k y \notin L$

i) $v = a^m (m > 0), x = b^l (l > 0) \Rightarrow vx \neq \epsilon, uv^k wx^k y = a^{n-mk} b^{n-lk} c^n \notin L$

ii) $v = b^m (m > 0), x = c^l (l > 0) \Rightarrow vx \neq \epsilon, uv^k wx^k y = a^n b^{n-mk} c^{n-lk} \notin L$

$\therefore L$ is not context-free

b) $L = \{x \in \{a, b, c\}^* \mid n_a(x) = \min\{n_b(x), n_c(x)\}\}$

b) $L = \{x \in \{a, b, c\}^* \mid n_a(x) = \min\{n_b(x), n_c(x)\}\}$

1) $\forall n \geq 0$

2) $\exists z = a^n b^n c^n \in L$ ($\because n = \min(n, n)$)

3) $\forall u, v, w, x, y: z = uvwx y, vx \neq \epsilon, |uvwx| \leq n$

i) $0 \leq \forall i \leq n: u = a^{n-i}, vwx = a^i b^{n-i}, y = b^i c^n$

ii) $0 \leq \forall i \leq n: u = a^n b^{n-i}, vwx = b^i c^{n-i}, y = c^i$

these two cases cover all the cases of z .

4) ... (same proof with above 1-a)

$\therefore L$ is not context-free

2. 문자열 "baaab"가 Text book(Example 7.34)의 언어 $L(G)$ 를 따르는지 **CYK algorithm**을 이용하여 보이시오. (3pt)

$$\begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned}$$

[CYK algorithm]

"baaab" $\in L(G)$

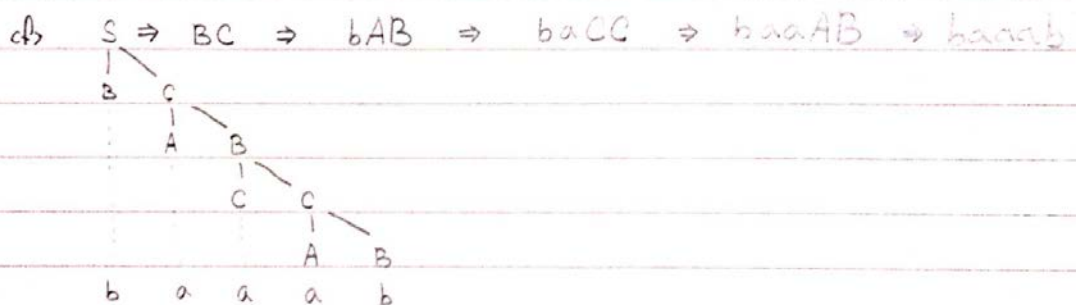
where G is

$$\left\{ \begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned} \right.$$

sol >

	b	a	a	a	b
len1	B	A,C	A,C	A,C	B
len2	A,S	B	B	S,C	
len3	-	A,S,C	B		
len4	S,C,A	S,C			
len5	S,C				

\therefore baaab $\in L(G)$



3. 다음 언어의 Turing machine을 그리시오. (3pt)

$$L = \{a^i b^i c^i \mid i \in \mathbb{N}\}$$

< Turing Machine >

$$L = \{a^i b^i c^i \mid i \in \mathbb{N}\}$$

equivalent to TM

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

$$\text{where } Q = \{S, B, C, X, \text{DEAD}, \text{END}\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{a, b, c, x, y, z, \varepsilon\}$$

$$\delta = \{S \xrightarrow{a/x, R} B, S \xrightarrow{b/y, R} \text{END},$$

$$B \xrightarrow{a/a, R} B, B \xrightarrow{b/y, R} C, B \xrightarrow{b/y, R} B$$

$$C \xrightarrow{b/b, R} C, C \xrightarrow{c/z, L} X, C \xrightarrow{z/\varepsilon, R} C$$

$$X \xrightarrow{z/\varepsilon, L} X, X \xrightarrow{b/b, L} X, X \xrightarrow{b/y, L} X,$$

$$X \xrightarrow{a/a, L} X, X \xrightarrow{x/x, R} S,$$

or 'all goes to (DEAD, ε , R) }

$$q_0 = S$$

$$B = \varepsilon$$

$$F = \{\text{END}\}$$

State 6, 7, 8 개 인 turning machine 중에서

(위의 정의에서는) X에서 합칠 수 있는 state 존재함을 나타낸 경우 정답처리