

①  $(A, \oplus)$  is an algebraic system, if  $\forall a, b \in A, a \oplus b \in A$ . closed.

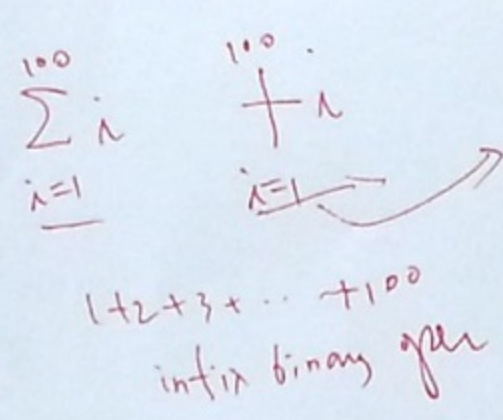


- Ex)  $(\mathbb{N}, +)$ ,  $(\mathbb{I}, -)$ ,  $(\mathbb{I}, \times)$ ,  $(\mathbb{I}, /)$ ,  $(\mathbb{R}, +)$ ,  $(\mathbb{R}, -)$ ,  $(\mathbb{R}, \times)$ ,  $(\mathbb{R}, /)$ ,  $\mathbb{R}, \sqrt{\quad}$ ,  $\mathbb{R}, \ln$ ,  $\frac{df(x)}{dx}$ ,  $\int f(x) dx$

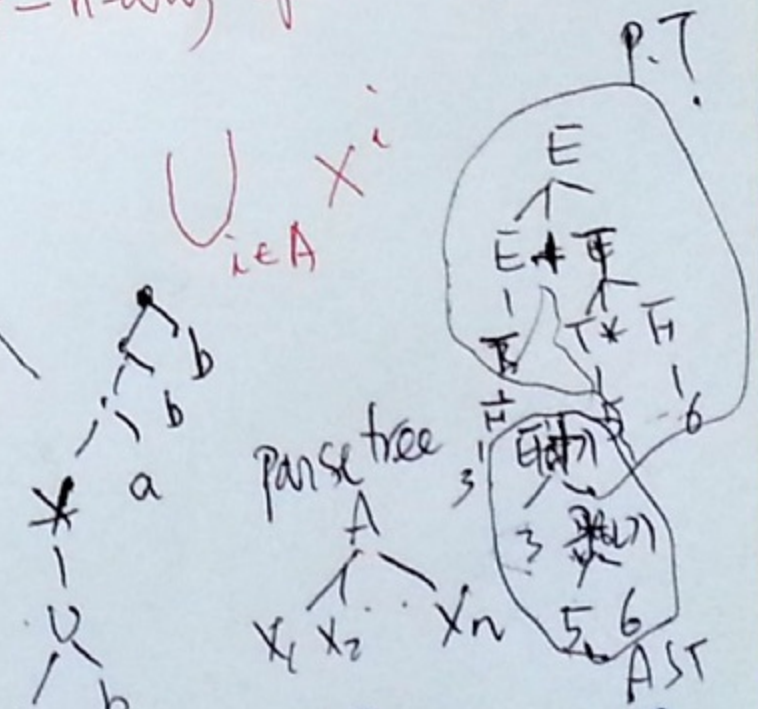
$(\mathbb{N}, +)$   
 $a + b = b + a$   
 $a + (b + c) = (a + b) + c$

P.T (NDS, P)  
 AST (N'US', P')  
 where  $N' \subseteq N, \Sigma' \subseteq \Sigma$   
 $P' \subseteq P$

②  $(A, \oplus)$  is an semigroup, if  
 1.  $\oplus$  is closed (=  $(A, +)$  is an algebraic system)  
 2.  $a \oplus (b \oplus c) = (a \oplus b) \oplus c$   $\oplus$  is associative.



100-ary prefix operator  
 $\bigcup_{i \in A} x_i$   
 Indexed set



③  $(A, \oplus, e)$  is an monoid, if  
 1.  $(A, +)$  is an semigroup.

2.  $e$  is an identity  $\exists \forall a \in A$   
 $a \oplus e = e \oplus a = a$

$(\mathbb{N}, +, 0)$  is an monoid.  
 $(\mathbb{N}, \times, 1)$  " " "

$(\Sigma^*, \cdot, \epsilon)$  " " "  
 $\Sigma$  .. a set of symbols  
 free string whose length is  $\geq 1$ .

$\forall x, y \in \Sigma^+ \quad x \cdot y \in \Sigma^+$   
 $(x \cdot y) \cdot z = x \cdot (y \cdot z)$   
 $x \cdot \epsilon = \epsilon \cdot x = x$   
 $\$ \$ = \$ 1 \$ 2 \dots \$ n$   
 $A \rightarrow x_1 x_2 \dots x_n$   
 $\rightarrow \{ \$ \$ \}$

lex yacc  
 L type 3 r.e (rg)  
 type 2 cfg.



3  
Minimization  
Define

$f: \mathbb{N}^n \rightarrow \mathbb{N}$  from  $g: \mathbb{N}^{n+1} \rightarrow \mathbb{N}$  by

$$f(\vec{x}) = \mu y [g(\vec{x}, y) = 0]$$

$\mu$ -Recursive (partial) functions  
is Turing computable

$\mu$ -RF  $\rightarrow$  TM      Thm 4.2 — easy  
TM  $\rightarrow$  MRFB      Thm 4.3