Chap. 6-1 Left and Right Parser

Redefinition *of a PDA* $P = (Q, \Sigma, \Gamma, \delta, q_0, \gamma_0, F)$ *is*

1. Q is a ***finite*** set of ***states***,
2. Σ is a ***finite*** set of ***input*** symbols,
3. Γ is a ***finite stack*** alphabet,
4. δ is a ***transition relation*** on ***configurations***,

$$\delta \subseteq (Q \times \Sigma^* \times \Gamma^*) \times (Q \times \Sigma^* \times \Gamma^*).$$

Let $q, p \in Q$, $x, y \in \Sigma^*$, $\alpha, \beta \in \Gamma^*$. We may write

$(q, x, \alpha) \rightarrow (p, \varepsilon, \beta)$ ***instead of*** $(p, \beta) \in \delta(q, x, \alpha)$.

If $(q, x, \alpha) \rightarrow (p, \varepsilon, \beta) \in \delta$, we may write

$(q, xy, \alpha\gamma) \Rightarrow_P (p, y, \beta\gamma)$ ***instead of*** $(q, xy, \alpha\gamma) \vdash_P (p, y, \beta\gamma)$.

5. $q_0 \in Q$ is an ***initial states***,
6. $\gamma_0 \in \Gamma^*$ is an ***initial stack content***,
7. $F \subseteq Q$ is a set of ***final states***.

$$L(P) = \{x \in \Sigma^* \mid (q_0, x, \gamma_0) \Rightarrow_P^* (f, \varepsilon, \gamma), f \in F, \gamma \in \Gamma^*\}.$$

Rewritng system $\rightarrow$ = (A, P) *where A is a set and* $P \subseteq A^* \times A^*$.

Let $\alpha, \beta, \gamma, \delta \in A^*$, $\alpha \rightarrow \beta \in P$. Then $\gamma\alpha\delta \Rightarrow_R \gamma\beta\delta$.

We say R is a rewriting system on A

R **rewrites** $\gamma\alpha\delta \in A^*$ to(derives) $\gamma\beta\delta \in A^*$.

We can **define** $\Rightarrow^i (\forall i \geq 0)$ and $\Rightarrow^*$.

A grammar $G = (N \cup \Sigma, P, S)$ is a **rewiting system** on $V^* = (N \cup \Sigma)^*$.

When we write $A \rightarrow_G \alpha$.

A FA $F = (Q, \Sigma, \delta, q_0, F)$ is a **rewiting system** on $Q \times \Sigma^*$.

When we write $(q, x) \rightarrow_F (p, \varepsilon)$ **instead of** $p \in \delta(q, x)$.

Old notation δ: $\delta^*(\delta(\delta^*(q_0, x), y), z) \in F$ considered **dirty**!

$(q_0, xyz) \Rightarrow_F^* (p, yz) \Rightarrow_{(p, y) \rightarrow (q, \varepsilon)} (q, z) \Rightarrow_F^* (f, \varepsilon)$ **consumer**

$A_{q_0} \Rightarrow_G^* xA_p \Rightarrow_{A_p \rightarrow yA_q} xyA_q \Rightarrow_G^* xyz$ **generator**

A PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ *is a* ***rewiting system*** *on* $(Q \times \Sigma^* \times \Gamma^*)$.

When we write $(q, x, \alpha) \rightarrow_P (p, \varepsilon, \beta)$ ***instead of*** $(p, \beta) \in \delta(q, x, \alpha)$.

If $(q, x, \alpha) \rightarrow_P (p, \varepsilon, \beta)$, *then* $(q, xy, \alpha\gamma) \Rightarrow_P (p, y, \beta\gamma)$.

Redefinition *of a PDA* $P = (\Gamma, \Sigma, \rightarrow, \iota, \Phi)$ *without states Q*

1. Γ *is a* ***finite stack*** *alphabet,*
2. Σ *is a* ***finite*** *set of* ***input*** *symbols,*
3. $\rightarrow$ *is a* ***rewriting*** *on* ***configurations***.

   $$\rightarrow \subseteq (\Gamma^* \times \Sigma^*) \times (\Gamma^* \times \Sigma^*).$$

   Let $\alpha, \beta \in \Gamma^*$, $x, y \in \Sigma^*$. *If* $(\alpha, x) \rightarrow (\beta, \varepsilon)$, *we may write*

   $$(\alpha\gamma, xy) \Rightarrow_P (\beta\gamma, y).$$

4. $\iota \in \Gamma^*$ *is an* ***initial stack content***, *and*
5. $\Phi \subseteq \Gamma^*$ *is a set of* ***final stack contents***.

$$L(P) = \{x \in \Sigma^* \mid (\iota, x) \Rightarrow_P^* (\phi, \varepsilon), \phi \in \Phi\}.$$

Left parser for a cfg $G = (N, \Sigma, P, S)$ *is a PDA* $L_P = (N \cup \Sigma, \Sigma, \rightarrow_L, \varepsilon, \{S\})$

$\forall A \rightarrow \alpha \in P$: $(A, \varepsilon) \rightarrow_L (\alpha, \varepsilon)$ *guess A as* $\alpha (A \rightarrow \alpha \in P)$.

$\forall a \in \Sigma$: $(a, a) \rightarrow_L (\varepsilon, \varepsilon)$ *verify* $a (a \in \Sigma)$.

Insight

$S \Rightarrow_{lm}^{*} xY\gamma \Rightarrow_{lm}^{*} xy\gamma \Rightarrow_{lm}^{*} xyz.$

case *1:* $Y = B \in N, B \rightarrow \beta \in P$.

$\bullet S \Rightarrow_{Lm}^{*} x \bullet B\gamma \Rightarrow_{Lm}^{B \rightarrow \beta} x \bullet \beta\gamma \Rightarrow_{Lm}^{*} xy \bullet \gamma \Rightarrow_{Lm}^{*} xyz \bullet.$

$(S, xyz) \Rightarrow_{L}^{*} (B\gamma, yz) \Rightarrow_{L}^{(B,\varepsilon) \rightarrow (\beta,\varepsilon)} (\beta\gamma, yz) \Rightarrow_{L}^{*} (\gamma, z) \Rightarrow_{L}^{*} (\varepsilon, \varepsilon).$

case *2:* $Y = a = y \in \Sigma, B \rightarrow \beta \in P$.

$\bullet S \Rightarrow_{Lm}^{*} x \bullet a\gamma \Rightarrow_{Lm}^{a} xa \bullet \gamma = xy \bullet \gamma \Rightarrow_{Lm}^{*} xyz \bullet.$

$(S, xyz) \Rightarrow_{L}^{*} (a\gamma, yz) = (a\gamma, az) \Rightarrow_{L}^{(a,a) \rightarrow (\varepsilon,\varepsilon)} (\gamma, z) \Rightarrow_{L}^{*} (\varepsilon, \varepsilon).$

Proof $A \Rightarrow_{lm}^{*} x\alpha$ *if and only if* $(A, x) \Rightarrow_{L}^{*} (\alpha, \varepsilon)$, $x \in \Sigma^{*}$, $\alpha \in (N \cup \Sigma)^{*}$.

(If) *If* $(A, x) \Rightarrow_{L}^{i} (\alpha, \varepsilon)$, *then* $A \Rightarrow_{lm}^{*} x\alpha$.

basis $i = 0$, $A = \alpha$, $x = \varepsilon$, *If* $(A, \varepsilon) \Rightarrow_{L}^{0} (A, \varepsilon) = (\alpha, \varepsilon)$, $A \Rightarrow_{lm}^{*} A = x\alpha$.

induction *Let* $i \geq 1$, *and consider the next-to-last step.*

i) $(\alpha, x) \Rightarrow_{L}^{i-1} (B\gamma, \varepsilon) \Rightarrow_{L} (\beta\gamma, \varepsilon) = (\alpha, \varepsilon)$

$\therefore A \Rightarrow_{lm}^{*} xB\gamma$ *by IH and* $B \rightarrow \beta \in P$ *by construction of* δ.

$\therefore A \Rightarrow_{lm}^{*} xB\gamma \Rightarrow_{lm} x\beta\gamma = x\alpha$.

ii) $(A, x) = (A, ya) \Rightarrow_{L}^{i-1} (a\alpha, a) \Rightarrow_{L} (\alpha, \varepsilon)$

$(A, y) \Rightarrow_{L}^{i-1} (a\alpha, \varepsilon)$ *by Thm 6.6*

$\therefore A \Rightarrow_{lm}^{*} ya\alpha = x\alpha$ *by IH*

(Only if) *If* $A \Rightarrow_{lm}^{i} x\alpha$, *then* $(A, x) \Rightarrow_{L}^{*} (q, \varepsilon, \alpha)$.

basis $i = 0$, $A \Rightarrow_{lm}^{0} A$. $(A, x) \vdash^{0} (q, \varepsilon, A)$.

induction *Let* $i \geq 1$*, and consider the next-to-last step.*

$A \Rightarrow_{lm}^{i-1} yB\gamma \Rightarrow_{lm} y\beta\gamma = y\underline{y'\gamma'}\gamma = x\alpha$ *where* $\beta = y'\gamma'$, $y' \in \Sigma^*$, $\gamma' \in (N \cup \Sigma)^*$.

$(A, y) \Rightarrow_L^* (B\gamma, \varepsilon)$ *by IH*, $(A, yy') \Rightarrow_L^* (B\gamma, y')$ *(by Thm 6.5)*

$\Rightarrow_L (\beta\gamma, y') = (y'\gamma'\gamma, y') \Rightarrow_L^{|y'|} (\gamma'\gamma, \varepsilon) = (\alpha, \varepsilon)$

$\therefore A \Rightarrow_{lm}^* x\alpha$ ***if and only if*** $(A, x) \Rightarrow_L^* (\alpha, \varepsilon)$.

If $A = S$, $\alpha = \varepsilon$, $S \Rightarrow_{lm}^{|\pi|} x$ ***if and only if*** $(S, x) \Rightarrow_L^{|\pi|+|x|} (\varepsilon, \varepsilon)$

$\therefore L(G) = L(L_P)$.

Right parser for a cfg $G = (N, \Sigma, P, S)$ *is a PDA* $R_P = (N \cup \Sigma, \Sigma \to_R, \varepsilon, \{S\})$

$\forall a \in \Sigma$:	$(\varepsilon, a) \to_R (a, \varepsilon)$	*shift* $a (a \in \Sigma)$
$\forall A \to \alpha \in P$:	$(\alpha^R, \varepsilon) \to_R (A, \varepsilon)$	*reduce* α *to* $A (A \to \alpha \in P)$.