

Example 2.6 $(0+1)^*$.

Example 2.7 The set of all binary strings beginning with prefix 01.

$01(0+1)^*$.

Example 2.8 The set of all binary strings having a substring 00.

$(0+1)^*00(0+1)^*$.

Example 2.9 The set of all binary strings having a substring 00101.

$(0+1)^*00101(0+1)^*$.

Example 2.10 The set of all binary strings ending with 00.

$(0+1)^*00$.

1. Binary strings **beginning** with $a_1 \dots a_k$. (**prefix** $a_1 \dots a_k$)

Consider $(k+2)$ states q_0, \dots, q_k , and d .

q_i stands for “prefix $a_1 \dots a_i$ of $a_1 \dots a_k$ is **found**”, $0 \leq \forall i \leq k$.

d stands for “found **no** prefix $a_1 \dots a_k$ ”

$0 \leq \forall i < k$,

$\forall a \in \Sigma, \delta(q_i, a) = q_{i+1}$, if $a = a_{i+1}$;

$\delta(q_i, a) = d$, if $a \neq a_{i+1}$ (otherwise).

$\forall a \in \Sigma, \delta(d, a) = d$. *dead state*

$\forall a \in \Sigma, \delta(q_k, a) = q_k$. *final state (prefix is already found)*

$M = (\{q_0, \dots, q_k, d\}, \Sigma, \delta, q_0, \{q_k\})$

2. Binary strings having a **substring** $a_1 \dots a_k$ (**substring** $a_1 \dots a_k$)

Consider $(k+1)$ states q_0, \dots, q_k

q_i stands for “prefix $a_1 \dots a_i$ of $a_1 \dots a_k$ is found”, $0 \leq \forall i \leq k$.

$$0 \leq \forall i < k,$$

$$\forall a \in \Sigma, \delta(q_i, a) = q_{i+1}, \text{ if } a = a_{i+1};$$

$$\delta(q_i, a) = q_j, \text{ if } a \neq a_{i+1} \text{ (otherwise).}$$

where j is the **maximum** index $\exists. a_1 \dots a_j = a_{i-j+1} \dots a_{i-1} a_i a$.

Note that $|a_{i-(j-1)} \dots a_{i-1} a_i a| = j-1 + 1 = j$.

$$\forall a \in \Sigma, \delta(q_k, a) = q_k \quad \text{final state (substring is already found)}$$

$$M = (\{q_0, \dots, q_k\}, \Sigma, \delta, q_0, \{q_k\})$$

3. Binary strings **ending** with $a_1 \dots a_k$. (suffix $a_1 \dots a_k$)

Consider $(k+1)$ states q_0, \dots, q_k with q_i standing for “found $a_1 \dots a_i$ ”

$$0 \leq \forall i \leq k,$$

$$\forall a \in \Sigma, \delta(q_i, a) = q_{i+1}, \text{ if } a = a_{i+1};$$

$$\delta(q_i, a) = q_j, \text{ if } a \neq a_{i+1} \text{ (otherwise).}$$

where j is the **maximum** index $\exists. a_1 \dots a_j = a_{i-j+1} \dots a_{i-1} a_i a$.

Note that $|a_{i-(j-1)} \dots a_{i-1} a_i a| = j$.

q_k final state (Suffix found but search for the next **suffix**, if any)

$$M = (\{q_0, \dots, q_k\}, \Sigma, \delta, q_0, \{q_k\})$$

Example 2.11 The set of binary expressions of positive integer which are congruent to m modulo k ($0 \leq m < k$).

Consider k -states q_0, \dots, q_{k-1} ,

q_i stands for “ $y \equiv i \pmod{k}$ ”, $0 \leq i < k$.

basis: $\delta(q_0, \varepsilon) = q_0$ and $\varepsilon \equiv 0 \pmod{k}$.

ind: Suppose $\delta(q_0, x) = q_i$ and $\delta(q_0, xa) = \delta(\delta(q_0, x), a) = \delta(q_i, a) = q_j$.

$j \equiv xa \pmod{k} \equiv 2i + a \pmod{k}$ for $a \in \{0, 1\}$

Final states = $\{q_m\}$

Special case $m=0$ and no leading zeros

new start state q_0' and dead state d ,

$\delta(q_0', 0) = d, \delta(q_0', 1) = q_{2*0+1 \pmod{k}} = q_{1 \pmod{k}} = q_1$.

$\delta(d, 0) = \delta(d, 1) = d$.

If no leading zeros except 0.

$\delta(q_0', 0) = f, \delta(q_0', 1) = q_1, \delta(f, 0) = \delta(f, 1) = d, \delta(d, 0) = \delta(d, 1) = d$.

Example 2.15 *The set of binary strings in which every block of four consecutive symbols contains a substring 01.*

Consider the complement \bar{L} contains a substring 0000, 1000, 1100, 1110 or 1111.