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14주-목 Chap 9. Undecidability

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TM ⋯ countably infinite ⋯ Σ^*

(program) = recursively enumerable ⋯ 자연수로 번호 매긴다.

└ 무한 자연수열

Σ^* ⋯ 〃 〃

TM 열

⋮

$M_i \in TM's$

$w_i \in \Sigma^*$

(M_i, w_i) pair $M_i \longleftrightarrow w_i$

Diagonalization Language vs Universal Lang.

$L_d = \{ w_i \in \Sigma^* \mid w_i \notin L(M_i) \}$

$L_u = \{ w_i \in \Sigma^* \mid w_i \in L(M_i) \}$

$= \overline{L_d}$

Ld is not R.E.

(proof) Assume Ld is R.E.

$\exists M \ \ni. \ Ld = L(M)$

$\exists i \in \mathbb{N}_0, \ M = M_i$

if $w_i \in Ld \rightarrow w_i \notin L(M_i) = Ld$ contradiction!

if $w_i \notin Ld \rightarrow w_i \in L(M_i) = Ld$

fi

∴ Ld is not R.E.

Not R.E. ⋯ Denial of Self recursion!

→ Cantor's diagonal arg.

- Halting problem
- Diagonalization Lang. Ld
- Russel's paradox $S = \{x \mid x \notin x\}$ $S \notin S \Leftrightarrow S \in S$, $p \Leftrightarrow \neg p$
- 이발사
- 모순형용사 (이음)
- 거짓말장이
- Gödel의 불완전성 원리 (G.I.T.)

Hilbert!

2) Undecidable that is RE.

T1, Recursive (decidable; algorithm) → dec.

if $x \in L \rightarrow M$ halts and accept.

□ $x \notin L \rightarrow M$ " " does not accept

fi

T0 R.E (computable; programmable) → Undecidable

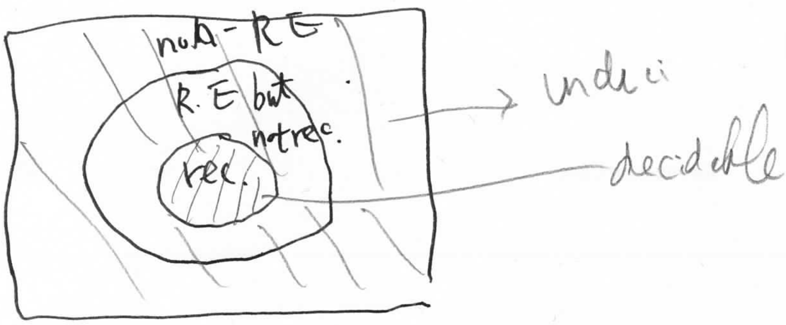
if $x \in L \rightarrow M$ halts and accept

□ $x \notin L \rightarrow M$ halts; ~~and~~ doesnot accept

OR M loopsforever.

fi

T-1 not R.E. (Non comp; Non prog.)



Languages vs. problems

$L : \Sigma^* \rightarrow \{0,1\} \iff P : D \rightarrow \{0,1\}$

언어와 문제는 같다

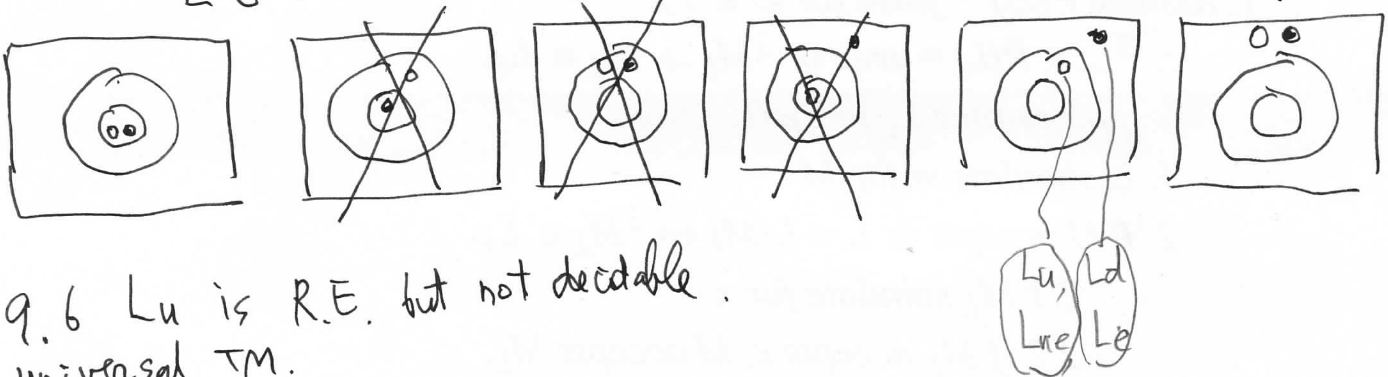
앞으로는 언어라는 말보다는 문제라는 말을 더 좋아한다.

3) Complement of problems.   ex) $L_d = \overline{L_u}$

Thm 9.3 If L is decidable, $\overline{L}$ is decidable

Thm 9.4 If L and $\overline{L}$ are R.E, L ($\overline{L}$) is decidable

L $\overline{L}$가 모두 RE이면 L, $\overline{L}$ 모두 RE not decidable은 없다?



Thm 9.6 L_u is R.E. but not decidable.

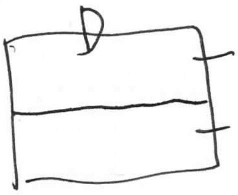
universal TM.

Simulate (M_i, w_i) pair

2) Problem and Reduction

$P: D \to \{0, 1\}$ D.. countable

$P(D)$... uncountable



yes-instance Y_p 1 (yes)

No-instance N_p 0 (no)

$P_1: D_1 \to \{0, 1\}$, $P_2: D_2 \to \{0, 1\}$

$P_1: D_1 \to \{0,1\}$ f $P_2: D_2 \to \{0,1\}$

문제 P_1이 P_2로 reduce 되었다고 부른다

P_2가 P_1보다 같거나 더 어렵다

P_2는 P_1보다 쉽지는 않다.

$f: D_1 \to D_2$ $P_1 \le_f P_2$

Thm 9.7. $P_1 \le P_2$

If P_1 is undec. → P_2 is undec (P_2 dec → P_1 dec)

" P_1 is non-R.E → P_2 " non-R.E. (P_2 RE → P_1 RE)

$L_e = \{M \mid L(M) = \emptyset\}$

① $L_{ne} = \{M \mid L(M) \neq \emptyset\}$ is R.E.

② $L_u \le L_{ne}$ → L_{ne} is R.E. but not dec.

③ L_e is not R.E.