Homework #1
CS322/KAIST 2011 Fall
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Exercise 1 (10pt) Prove that {ㄱ, ㄴ, ⋯, ㅎ, ㄲ, ㄸ, ⋯, ㅉ, ㅏ, ㅑ, ⋯, ㅣ}* is countable.

→ Let Σ = {ㄱ, ㄴ, ⋯, ㅎ, ㄲ, ㄸ, ⋯, ㅉ, ㅏ, ㅑ, ⋯, ㅣ}.
See **Canonical(lexicographical) order for** Σ^* in TP, [Supplementary TP1] Reviews on Discrete Mathmatics, page 20.

Exercise 2 (10pt)
a) (4pt) Show that

$$n! > 2^n \quad \text{for } n \geq 4,\ n \in N$$

→
1) basis: $n=4,\ 4! \geq 2^4$ (true)
2) induction: Assume that the statement holds when $n=k$
$k! > 2^k \qquad \times (k+1)$
$k!(k+1) > 2^k(k+1)$
$(k+1)! > 2^k(k+1) > 2 \times 2^k \qquad (k+1 > 2,\ \text{for } k > 4)$
$(k+1)! > 2^{(k+1)}$
The statement also holds when $n=k+1$.

By mathematical induction, $n! > 2^n$ for $n \geq 4,\ n \in N$.

a) (6pt) According to an ancient legend, there lies three diamond rods on the top of a high and high mountain. At the beginning of time, a hermit has put 64 golden discs on the leftmost rod. The size of each disc are different, where the discs lie at the rod in descending order. It means that the lowest disc is largest and the highest disc is smallest. Everyday, the hermit moves the discs, in accordance with following rules:

1. Only one disc can be moved at a time
2. Larger disc cannot be placed on top of smaller disc

When he moves all of the disc to the rightmost rod, it is said that the world will end. Prove that the (minimal) number of moves is $2^{64}-1$ which means that it is too early to worry about the end of the world!

→

We show that the number of moves is 2^n-1 where n is the number of discs.

1) basis: $n=1$, $2^1-1=1$. We only need to move single disc. (true)
2) induction: Assume that the statement holds when $n=k$
In order to move $k+1$ discs, we need to i) move top k discs to temporal (empty) rod, ii) move largest disc to target rod, and then iii) move all (k) discs to target rod.
The number of moves is $(2^k-1)+1+(2^k-1)=2\times 2^k-1=2^{k+1}-1$.
The statement also holds when $n=k+1$.

By mathematical induction, the number of moves is 2^n-1.