Example 2.6 *$(0+1)^*$.*

Example 2.7 *The set of all binary strings beginning with prefix 01.*

$01(0+1)^*$.

Example 2.8 *The set of all binary strings having a substring 00.*

$(0+1)^*00(0+1)^*$.

Example 2.9 *The set of all binary strings having a substring 00101.*

$(0+1)^*00101(0+1)^*$.

Example 2.10 *The set of all binary strings ending with 00.*

$(0+1)^*00$.

*1. Binary strings **begining** with $a_1 \ldots a_k$.* ***(prefix** $a_1 \ldots a_k$**)***

Consider (k+2) states $q_0, \ldots, q_k$, and d.

q_i *stands for "prefix $a_1 \ldots a_i$ of $a_1 \ldots a_k$ is **found**", $0 \le {}^\forall i \le k$.*

*d stands for "found **no** prefix $a_1 \ldots a_k$"*

$0 \le {}^\forall i < k,$

$${}^\forall a \in \Sigma,\ \delta(q_i, a) = q_{i+1}, \text{ if } a = a_{i+1};$$

$$\delta(q_i, a) = d, \text{ if } a \ne a_{i+1} \text{(otherwise)}.$$

${}^\forall a \in \Sigma,\ \delta(d, a) = d.$ *dead state*

${}^\forall a \in \Sigma,\ \delta(q_k, a) = q_k.$ *final state(**prefix** is already found)*

$$M = (\{q_0, \ldots, q_k, d\}, \Sigma, \delta, q_0, \{q_k\})$$

2. Binary strings having a ***substring*** $a_1 \ldots a_k$. (***substring*** $a_1 \ldots a_k$)

Consider (k+1) states $q_0, \ldots, q_k$

q_i *stands for "prefix* $a_1 \ldots a_i$ *of* $a_1 \ldots a_k$ *is* ***found***", $0 \le {}^\forall i \le k$.

$0 \le {}^\forall i < k$,

${}^\forall a \in \Sigma$, $\delta(q_i, a) = q_{i+1}$, *if* $a = a_{i+1}$;

$\delta(q_i, a) = q_j$, *if* $a \ne a_{i+1}$*(otherwise).*

where j is the ***maximum*** *index* .∍. $a_1 \ldots a_j = a_{i-j+1} \ldots a_{i-1} a_i \mathbf{a}$.

Note that $|a_{i-(j-1)} \ldots a_{i-1} a_i a| = j-1 + 1 = j$.

${}^\forall a \in \Sigma$, $\delta(q_k, a) = q_k$. *final state(****substring*** *is already found)*

$M = (\{q_0, \ldots, q_k\}, \Sigma, \delta, q_0, \{q_k\})$

*3. Binary strings **ending** with $a_1 \ldots a_k$.* *(**suffix** $a_1 \ldots a_k$)*

Consider (k+1) states $q_0, \ldots, q_k$ with q_i standing for "found $a_1 \ldots a_i$"

$0 \leq {}^{\forall}i \leq k$,

${}^{\forall}a \in \Sigma$, $\delta(q_i, a) = q_{i+1}$, if $a = a_{i+1}$;

$\delta(q_i, a) = q_j$, if $a \neq a_{i+1}$(otherwise).

*where j is the **maximum** index* .϶. $a_1 \ldots a_j = a_{i-j+1} \ldots a_{i-1}a_i\mathbf{a}$.

Note that $|a_{i-(j-1)} \ldots a_{i-1}a_i a| = j$.

q_k. *final state(Suffix found but search for the next **suffix**, if any)*

$M = (\{q_0, \ldots, q_k\}, \Sigma, \delta, q_0, \{q_k\})$

Example 2.11 *The set of binary expressions of positive integer which are congruent to m modulo k*$(0 \le m < k)$.

Consider k-states $q_0, \ldots, q_{k-1}$,

q_i *stands for* "$y \equiv i \pmod{k}$", $0 \le i < k$.

basis: $\delta(q_0, \varepsilon) = q_0$ *and* $\varepsilon \equiv 0 \pmod{k}$.

ind: Suppose $\delta(q_0, x) = q_i$ *and* $\delta(q_0, xa) = \delta(\delta(q_0, x), a) = \delta(q_i, a) = q_j$.

$j \equiv xa \pmod{k} \equiv 2i + a \pmod{k}$ *for* $a \in \{0, 1\}$

Final states $= \{q_m\}$

Special case m=0 and no leading zeros

new start state q_0' *and dead state d,*

$\delta(q_0', 0) = d$, $\delta(q_0', 1) = q_{2*0+1 \pmod{k}} = q_{1 \pmod{k}} = q_1$.

$\delta(d, 0) = \delta(d, 1) = d$.

If no leading zeros except 0.

$\delta(q_0', 0) = f$, $\delta(q_0', 1) = q_1$, $\delta(f, 0) = \delta(f, 1) = d$, $\delta(d, 0) = \delta(d, 1) = d$.

Example 2.15 *The set of binary strung in which every block of four consequtive symbols contains a substring 01.*

Consider the complemet $\overline{L}$ contains a substring 0000, 1000, 1100, 1110 or 1111.