Chap. 2 Finite Automata

2.1 An Informal Picture of Finite Automata

A man with a wolf, goat, and cabbage is on the left bank of a river

A boat carries one man and only one of the other three.

The wolf eats the goat without the man.

The goat eats the cabbage without the man.

Is it possible to cross the river without the goat or cabbage being eaten?

***states**: occupants of each bank after a crossing*

16 subsets of the man(m), wolf(W), goat(G), and cabbage(C).

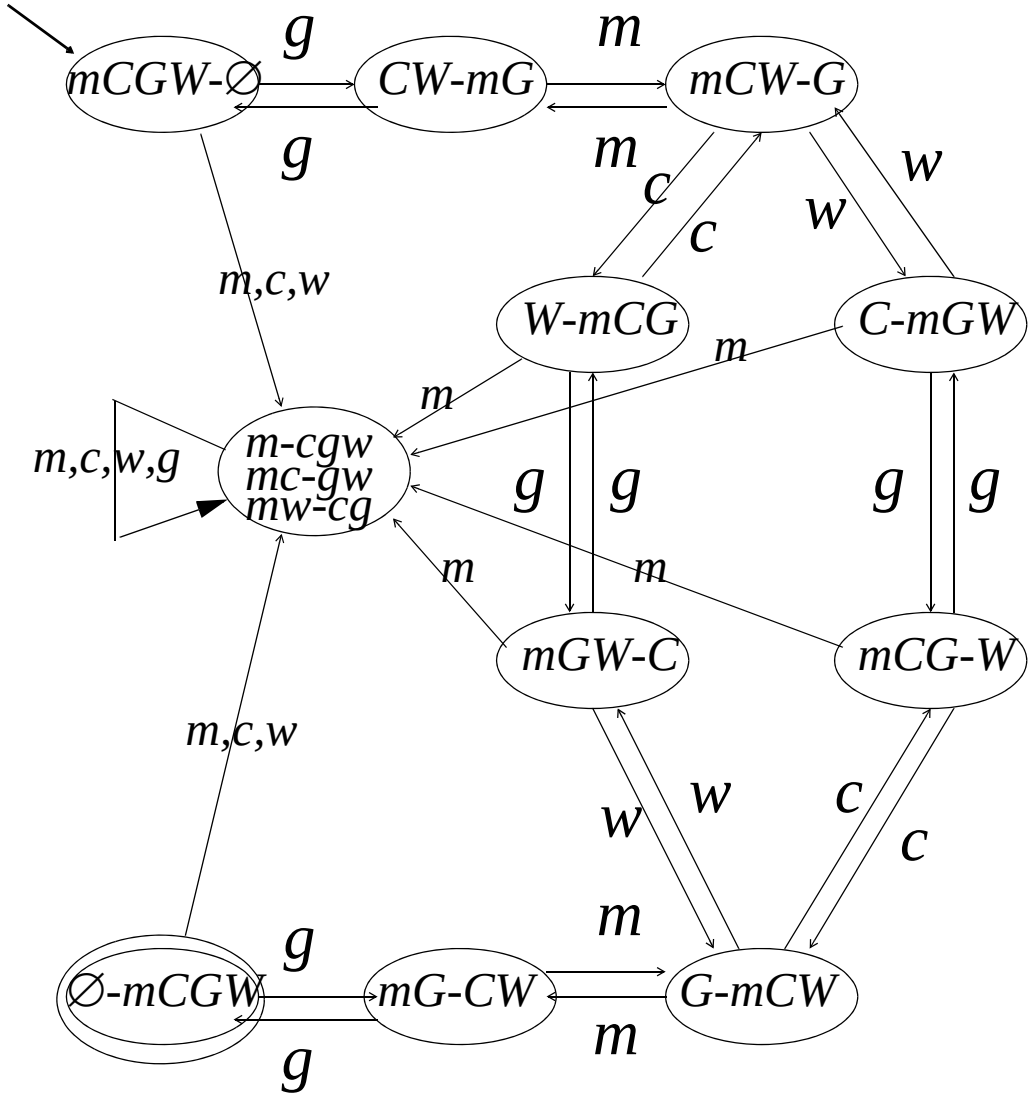
16 states: mWGC-∅, WC-mG, …, ∅-mWGC

***input symbol**: four ways of crossing the river*

man cross alone(m), with the wolf(w), the goat(g), or cabbage(c)

***input string**: a **sequence** of crossings(input symbols)*

state transition**: From a **state** to another **state** by a **input symbol


mCGW-∅
CW-mG
mCW-G
W-mCG
C-mGW
m-cgw
mc-gw
mw-cg
m,c,w,g
mGW-C
mCG-W
∅-mCGW
mG-CW
G-mCW
g
m
c
w
m,c,w

2.2 Deterministic Finite Automata

A **deterministic finite automaton** consists of:

1. A **finite** set of **states**, denoted Q,
2. A **finite** set of **input symbols**, denoted Σ,
3. A **transition function**,

   $\delta: Q \times \Sigma \rightarrow Q$

   Let $p, q \in Q$, and $a \in \Sigma$, if we write $\delta(q, a) = p$, meaning that
   when **current** state is in q and **input** symbol is a,
   the current state is changed to p, and
   input symbol is **changed** to the **next(right)** one.
4. A **start(initial)** state, $q_0 \in Q$, and
5. A set of **final** or **accepting** states, $F \subseteq Q$.

A DFA $A = (Q, \Sigma, \delta, q_0, F)$ "five-tuple" notation

2.2.2 How a DFA Processes Strings

*How a DFA decides whether or not "**accept**"*

*a **input string(sequence of input symbols)***

Suppose $a_1a_2 \ldots a_n$ is a sequence of input symbols

*1. We start out the DFA with in its **start** state q_0.*

*2. We consult the **transition function**, δ,*

with current state and current input symbol,

asumme in the first trial $\delta(q_0, a_1) = q_1$,

*3. **Repeat** this step with the **next** state q_1 and the **next** input symbol a_2.*

Assume $\delta(q_1, a_2) = q_2$, $\delta(q_2, a_3) = q_3$, …, $\delta(q_{n-1}, a_n) = q_n$.

*4. **Finally**, check if $\delta(q_{n-1}, a_n) = q_n \in F$ or not.*

*If $q_n \in F$, **accept** $a_1a_2 \ldots a_n$. If $q_n \notin F$ **does not accept** $a_1a_2 \ldots a_n$.*

$\delta(\delta(\ldots\delta(\delta(q_0, a_1), a_2), \ldots), a_{n-1}), a_n) = q_n$

where $\delta(q_{i-1}, a_i) = q_i$ for $1 \leq {}^{\forall} i \leq n$.

Example *2.1 Let* $\Sigma = \{0, 1\}$ *and* $L = \{x01y \in \Sigma^* \mid x, y \in \Sigma^*\}$.

State q_0: ***initial*** *state*

If it sees 0, go to a ***new*** *state* q_1. *If it sees 1* ***stay*** *in the state* q_0.

It has ***never*** *seen 0.*

State q_1: *The* ***last symbol*** *it has seen is 0.*

If it sees 1, go to a ***new*** *state* q_2. *If it sees 0,* ***stay*** *in the state* q_1.

State q_2: *It has* ***ever*** *seen is 01.* ***final*** *state*

Whether it sees 0 or 1, stay in the state q_2.

$A = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2\}$ $\Sigma = \{0, 1\}$

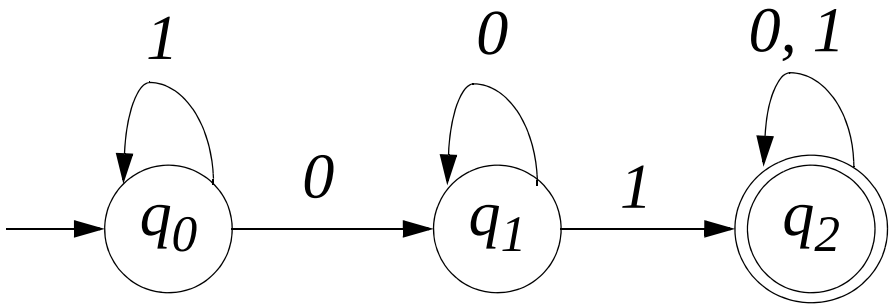
$\delta = \{\delta(q_0, 0)=q_1, \delta(q_0, 1)=q_0, \delta(q_1, 0)=q_1, \delta(q_1, 1)=q_2,$

$\delta(q_2, 0)=q_2, \delta(q_2, 1)=q_2\}$

$F = \{q_2\}$

2.2.3 Simple notations for DFA's

Transition diagram



Transition table

		0	1
→	q_0	q_1	q_0
	q_1	q_1	q_2
*	q_2	q_2	q_2

2.2.4 Extend the domain of transition function from symbols to strings

$\delta: Q \times \Sigma \to Q$

$\hat{\delta}: Q \times \Sigma^* \to Q$

$\hat{\delta}(q, \varepsilon) = q$ **basis**

$\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$ **recursion**

$q \in Q, w \in \Sigma^*, a \in \Sigma, wa \in \Sigma^+.$

$\hat{\delta}(q_0, a_1a_2 \ldots a_n) = \delta(\hat{\delta}(q_0, a_1a_2 \ldots a_{n-1}), a_n)$ *1st* **recursion**

$= \delta(\delta(\hat{\delta}(q_0, a_1a_2 \ldots a_{n-2}), a_{n-1}), a_n)$ *2nd* **recursion**

$= \ldots$

$= \delta(\delta(\ldots\delta(\hat{\delta}(q_0, a_1), a_2), \ldots), a_{n-1}), a_n)$ *(n-1)-th* **recursion**

$= \delta(\delta(\ldots\delta(\delta(\hat{\delta}(q_0, \varepsilon), a_1), a_2), \ldots), a_{n-1}), a_n)$ *n-th* **recursion**

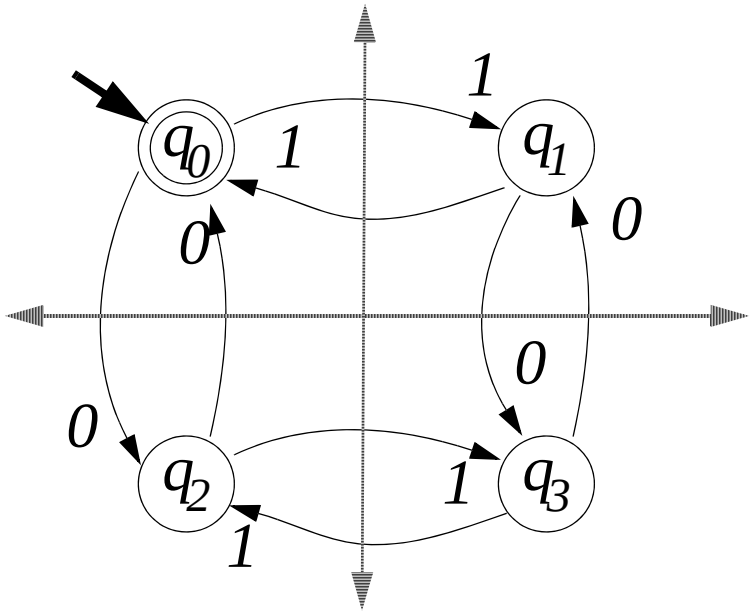
$= \delta(\delta(\ldots\delta(\delta(q_0, a_1), a_2), \ldots), a_{n-1}), a_n)$ **basis**

If $\hat{\delta}(q, w) = p$, for some $q, p \in Q$ and $w \in \Sigma^*$,

*w is the **path** from the state q to the state p.*

Since $\hat{\delta} \supseteq \delta$, we may use δ instead of $\hat{\delta}$.

Example 2.4



even numbers of 0's and even numbers of 1's.

2.2.5 The Language of DFA

*We **define** the language of a DFA $A = (Q, \Sigma, \delta, q_0, F)$, $L(A)$,*

$$L(A) = \{w \in \Sigma^* | \hat{\delta}(q_0, w) \in F\} \text{ or } \{w \in \Sigma^* | \delta(q_0, w) \in F\}, \text{ for short.}$$

$$L(A) \subseteq \Sigma^* \text{ or } L(A) \in 2^{\Sigma^*}.$$

Definition: Regular languages

Let L be a language.

*If L = L(A) for some **DFA** A, then we say L is a **regular language**.*

*a **class of languages***

***type-3 languages** in Chomsky's language hierarchy*

Definition: Equivalence of Automata

*Let M_1 and M_2 be two automata. We say two automata M_1 and M_2 are **equivalent**, if $L(M_1) = L(M_2)$.*

*The **state** of automata*

__summarizes__ the information concerning __past inputs__

that is needed to __determine__ the __behavior__ of the automata

on __subsequent inputs__.

Aho and Ullman

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and $q \in Q$.

We define __accessible__ input strings from initial state to the state q.

$$L(q) = \{w \in \Sigma^* | \ \hat{\delta}(q_0, w) = q\}$$

$$L(M) = \cup_{q \in F} L(q).$$

Consider a __binary relation__ R_M on Σ^* and $x \, R_M \, y$, if $\hat{\delta}(q_0, x) = \hat{\delta}(q_0, y)$.

Then R_M is a __quivalent__ relation and

the __equivalent class__ $[x]_{R_M} = L(q)$, if $\hat{\delta}(q_0, x) = q$.

Since R_A is equivalent, $[x]_{R_M}$ is a __partion__ of Σ^*.

2.3 Nondeterministic Finite Automaton

$A = (Q, \Sigma, \delta, q_0, F)$ is a ***nondeterministic finite automaton****(NFA), if*

1. *Q is a* ***finite*** *set of* ***states****,*
2. *Σ is a* ***finite*** *set of* ***input symbols****, open denoted,*
3. *δ is a* ***transition function****,*

   $\delta: Q \times \Sigma \rightarrow 2^Q$.

   If $q \in Q$, $a \in \Sigma$, and $\{p_1, p_2, \ldots, p_n\} \subseteq Q$(or $1 \leq {}^{\forall} i \leq n, p_i \in Q$)
   then we write $\delta(q, a) = \{p_1, p_2, \ldots, p_n\}$, meaning that
   when current state is in q and input symbol is a,
   the state may be changed to ***any*** *one of $p_1, p_2, \ldots, p_n$,*
   and input symbol is changed to the next(right) one.
4. *$q_0 \in Q$ is a* ***start*** *state, and*
5. *$F \subseteq Q$ is a set of* ***final*** *or* ***accepting*** *states.*

   Fig. 2.9, 2.10, 2.11

DFA: $\delta: Q \times \Sigma \to Q$

$^{\forall}q \in Q, ^{\forall} a \in \Sigma, ^{\exists}\delta(q, a) \in Q.$ *(**total**) function*

Otherwise ***partial*** *function*

If $A = (Q, \Sigma, \delta, q_0, F)$ *be a FA with* $\delta: Q \times \Sigma \not\to Q$ ***partial*** *function,*

$L(A) = L(B)$ *for DFA* $B = (Q \cup \{d\}, \Sigma_1, \delta', q_0, F)$ *where* $d \notin Q$,

$\delta' = \delta \cup \{\delta(q, a) = d | \delta(q, a) \notin Q\} \cup \{\delta(d, a) = d | a \in \Sigma\}$

d: ***dead*** *state*

B is the DFA(with total function)

DFA:	$\delta: Q \times \Sigma \to Q.$
DFA but partial function:	$\delta: Q \times \Sigma \to Q \cup \{\varnothing\}.$
NFA:	$\delta: Q \times \Sigma \to 2^Q.$

2.3.3 The Extended Transition Function

DFA

$\delta: Q \times \Sigma \to Q$

$\hat{\delta}: Q \times \Sigma^* \to Q$

NFA

$\delta: Q \times \Sigma \to 2^Q$

$\delta': 2^Q \times \Sigma \to 2^Q$ *in this lecture*

$\hat{\delta}': 2^Q \times \Sigma^* \to 2^Q$ *in this lecture* $\quad \hat{\delta}: Q \times \Sigma^* \to 2^Q$ *in the text*

Two step extension

1. Set extension: $\delta \Rightarrow \delta'$

We extend the 1st domain of δ *to* ***set of states***.

$\delta': 2^Q \times \Sigma \to 2^Q$.

Let $P \subseteq Q$, *we define*

$$\delta'(P, a) = \{q \in Q | \ p \in P, q \in \delta(p, a)\}$$

$or \quad = \cup_{p \in P} \delta(p, a)$

We may write $\delta'(p, a)$ ***instead*** *of* $\delta'(\{p\}, a)$ *when needed.*

2. String extension *:* $\delta' \Rightarrow \hat{\delta}$*(same as* $\delta \Rightarrow \hat{\delta}$*)*

$\hat{\delta}: 2^Q \times \Sigma^* \rightarrow 2^Q$.

$\hat{\delta}(P, \varepsilon) = P$ ***basis***

$\hat{\delta}(P, wa) = \delta'(\hat{\delta}(P, w), a)$ ***recursion***

$P \subseteq Q (or\ P \in 2^Q)$, $w \in \Sigma^*$, $a \in \Sigma$, $wa \in \Sigma^+$.

We may use δ *instead of* δ' *or* $\hat{\delta}$*, since* $\delta \subseteq \delta', \hat{\delta}$.

Let $N = (Q, \Sigma, \delta, q_0, F)$ *be an NFA. Then the* ***language defined*** *by the NFA N, denoted as L(N) is,*

$L(N) = \{w \in \Sigma^* | \ \hat{\delta}(\{q_0\}, w) \cap F \neq \varnothing\}$.

$L(N) \subseteq \Sigma^*$ *or* $L(N) \in 2^{\Sigma^*}$.

2.3.5 Equivalence of DFA and NFA

Example 2.10(Fig. 2.12)

Subset construction*(NFA ⇒ DFA)*

Given an NFA $N = (Q_N, \Sigma, \delta_N, q_{0N}, F_N)$,

construct *DFA* $D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$.

1. $Q_D = \{P | P \subseteq Q_N\} = 2^{Q_N}$.

3. $\delta_D(P, a) = \{q \in Q_N | p \in P \subseteq Q_N, q \in \delta_N(p, a)\}$

   or $= \cup_{p \in P} \delta_N(p, a)$

   or $= \delta_N'(P, a)$ ***set extension*** *of* δ_N.

   $\delta_D(P, a) = \delta_N'(P, a)$ *or* $\delta_D = \delta_N'$ *in short,*

where $\delta_D: Q_D \times \Sigma \rightarrow Q_D$, $\delta_N': 2^{Q_N} \times \Sigma \rightarrow 2^{Q_N}$, *and* $Q_D = 2^{Q_N}$.

4. $q_{0D} = \{q_{0N}\}$.

5. $F_D = \{F \in Q_D | F \cap F_N \neq \emptyset\}$

Theorem *2.11 L(D) = L(N) in the above subset construction*

Proof $\hat{\delta}_D(q_{0D}, w) = \hat{\delta}'_N(\{q_{0N}\}, w)$ *for all* $w \in \Sigma^*$.

$\hat{\delta}_D$: $Q_D \times \Sigma^* \rightarrow Q_D = 2^{Q_N} \times \Sigma^* \rightarrow 2^{Q_N}$.

$\hat{\delta}'_N$: $2^{Q_N} \times \Sigma^* \rightarrow 2^{Q_N}$. *We may use* q_{0N} *instead of* $\{q_{0N}\}$.

Induction *on* $|w|$.

basis: *Let* $|w| = 0$, $\hat{\delta}_D(q_{0D}, \varepsilon) = \hat{\delta}_D(\{q_{0N}\}, \varepsilon) = \{q_{0N}\} = \hat{\delta}'_N(\{q_{0N}\}, \varepsilon)$.

induction: *Let* $w = xa$ *where* $a \in \Sigma$, $x \in \Sigma^*$, *and* $w \in \Sigma^+$. $(|w| = |x| + 1)$

$\hat{\delta}_D(q_{0D}, xa) = \delta_D(\hat{\delta}_D(\{q_{0N}\}, x), a)$ *by definition of* $\hat{\delta}_D$.

$= \delta_N'(\hat{\delta}_D(\{q_{0N}\}, x), a)$ *by* ***subset construction***

$= \delta_N'(\hat{\delta}'_N(\{q_{0N}\}, x), a)$ *by induction hypothesis*

$= \hat{\delta}'_N(\{q_{0N}\}, xa)$ *by definition of* $\hat{\delta}'_N$.

$\hat{\delta}_D(\{q_0\}, x) \in F_D$, *if and only if,* $\hat{\delta}_N(q_0, x) \cap F_N \neq \emptyset$.
Q.E.D.

Theorem 2.12 *A language L is defined by some DFA, if and only if L is defined by some NFA.(NFA* $\Leftrightarrow$ *DFA)*
Proof*: (If) subset construction(NFA* $\Rightarrow$ *DFA)*
(Only if) NFA can ***easily simulate*** *DFA(DFA* $\Rightarrow$ *NFA).*

Let a DFA $D = (Q, \Sigma, \delta_D, q_0, F)$. *We define an NFA*$(Q, \Sigma, \delta_N, q_0, F)$

$$\delta_N(q, a) = \{p \mid \delta_D(q, a) = p\}.$$

Proof *for* $^{\forall}w \in \Sigma^*$, *if and only if,* $\hat{\delta}_D(q, w) = p$, *then* $\hat{\delta}_D(q, w) = \{p\}$.
Then L(D) = L(N).

$\therefore$ *L is* ***regular****, iff L = L(N) for some* ***NFA*** *N.*

2.5 Finite Automata with Epsilon-Transitions

$A = (Q, \Sigma, \delta, q_0, F)$ is a ***fa with epsilon transition*** *(ε-NFA), if*

Q, Σ, q_0, and F are same as NFA, but

3. δ is a ***transition function****,*

$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow 2^Q$.

If $q \in Q$, and $\{p_1, p_2, \ldots, p_n\} \subseteq Q$,

then we write $\delta(q, \varepsilon) = \{p_1, p_2, \ldots, p_n\}$, meaning that

when current state is in q and ***regardless*** *of input symbol*

the state may be changed to ***any*** *one of $p_1, p_2, \ldots, p_n$,*

and next input symbol is ***not changed****.*

Ex. 2.16, Fig. 2.18

2.5.3 Epsilon-closure

basis: $q \in \varepsilon^*(q)$ (**same as** *ECLOSE(q) in the text)*

recursion: *If* $p \in \varepsilon^*(q)$, *and* $r \in \delta(p, \varepsilon)$, *then* $r \in \varepsilon^*(q)$.

Example 2.18

Why $\varepsilon^*(q)$ ***instead*** *of ECLOSE(q)*

Let $\varepsilon = \{(p, q) | \, q \in \delta(p, \varepsilon)\}$. *Then* $\varepsilon \subseteq Q \times Q$ *(a binary relation on Q) and ECLOSE(q)* $= \varepsilon^*(q)$ ***reflexive-transitive*** *closure of* ε.

2.5.4 The Extended Transition and Language for ε-NFA's

NFA: δ*:* $Q \times \Sigma \to 2^Q$.

ε*-NFA:* δ*:* $Q \times (\{\varepsilon\} \cup \Sigma) \to 2^Q$.

Let's ***divide*** δ *of* ε*-NFA into* δ^1 *and* δ^0.

δ^1*:* $Q \times \Sigma \to 2^Q$.

δ^0*:* $Q \times \{\varepsilon\} \to 2^Q$.

$\delta = \delta^1 \cup \delta^0$.

δ^1*: is* ***same*** *as* δ *in NFA, we use* δ ***instead*** *of* δ^1*, since* $\delta^1 \subseteq \delta$.

$\delta^0 \leftrightarrow_f \varepsilon$ *what is the* ***bijection*** *f? We use* ε ***instead*** *of* δ^0.

$\varepsilon = \{(p, q) | \, q \in \delta(p, \varepsilon)\} \subseteq Q \times Q$ *binary* ***relation*** *on Q.*

$\varepsilon^* = \cup_{i \in N_0} \varepsilon^i \subseteq Q \times Q$ *binary* ***relation*** *on Q.*

$\varepsilon, \varepsilon^*$*:* $Q \to 2^Q$. ***function*** *from Q to* 2^Q.

*Let's **extend** the domain of $\delta(=\delta^1)$, ε, and ε^* from **state** to **set of states**.*
Let $P \subseteq Q$, we define

δ': $2^Q \times \Sigma \to 2^Q$.

$\delta'(P, a) = \{q \in Q \mid p \in P, \delta(p, a) = q\}$

or $= \cup_{p \in P} \delta(p, a)$

*We may write $\delta'(p, a)$ **instead** of $\delta'(\{p\}, a)$ when needed.*

*We may write $\delta(P, a)$ **instead** of $\delta'(P, a)$ when needed.*

ε': $2^Q \to 2^Q$.

$\varepsilon'(P) = \{q \in Q \mid p \in P, \varepsilon(p) = q\}$

or $= \cup_{p \in P} \varepsilon(p)$

ε'^*: $2^Q \to 2^Q$.

$\varepsilon'^*(P) = \{q \in Q \mid p \in P, \varepsilon^*(p) = q\} \quad = \cup_{p \in P} \varepsilon^*(p)$

*We write $\varepsilon(P)$ **instead** of $\varepsilon'(P)$ and $\varepsilon^*(P)$ **instead** of $\varepsilon'^*(P)$.*

*Let's **extend** the of domain of δ from **symbol** to **strings**.*

$\hat{\delta}: 2^Q \times \Sigma^* \to 2^Q$.

$\hat{\delta}(P, \varepsilon) = \varepsilon^*(P)$ ***basis***

$\hat{\delta}(P, wa) = \varepsilon^*(\delta(\hat{\delta}(P, w), a))$ ***recursion***

$P \subseteq Q (or\ P \in 2^Q), w \in \Sigma^*, a \in \Sigma, wa \in \Sigma^+$.

$\hat{\delta}(q_0, a_1a_2 \ldots a_n) = \varepsilon^*(\delta(\hat{\delta}(q_0, a_1a_2 \ldots a_{n-1}), a_n))$ *1st* ***recursion***

$= \varepsilon^*(\delta(\varepsilon^*(\delta(\hat{\delta}(q_0, a_1a_2 \ldots a_{n-2}), a_{n-1})), a_n))$ *2nd* ***recursion***

$= \ldots$

$= \varepsilon^*(\delta(\varepsilon^*(\delta(\ldots\varepsilon^*(\delta(\hat{\delta}(q_0, a_1), a_2)), \ldots), a_{n-1})), a_n))$*(n-1)th* ***recursion***

$= \varepsilon^*(\delta(\varepsilon^*(\delta(\ldots\varepsilon^*(\delta(\varepsilon^*(\delta(\hat{\delta}(q_0, \varepsilon), a_1)), a_2)), \ldots)), a_{n-1})), a_n))$ ***n-th***

$= \varepsilon^*(\delta(\varepsilon^*(\delta(\ldots\varepsilon^*(\delta(\varepsilon^*(\delta(\varepsilon^*(q_0), a_1)), a_2)), \ldots)), a_{n-1})), a_n))$ ***basis***

$\delta^n = \varepsilon^* \circ \delta \circ \varepsilon^* \circ \ldots \circ \delta \circ \varepsilon^* = \varepsilon^* \circ (\delta \circ \varepsilon^*)^n$.

We may write δ^* ***instead*** *of* $\hat{\delta}$, *since* $\hat{\delta} = \cup_{i \in N_0} \delta^i = \delta^*$.

Let $E = (Q, \Sigma, \delta, q_0, F)$ *be an* ε*-NFA. Then*

the ***language defined by*** *the* ε*-NFA E, denoted as L(E) is,*

$$L(E) = \{w \in \Sigma^* | \ \hat{\delta}(q_0, w) \cap F \neq \varnothing\}.$$

$$L(E) \subseteq \Sigma^* \text{ or } L(E) \in 2^{\Sigma^*}$$

2.5.5 Eliminating ε-Transitions

Given an ε-NFA $E = (Q_E, \Sigma, \delta_E, q_0, F_E)$*, construct the*

equivalent *DFA* $D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$ *such that* $L(D) = L(N)$.

1. $Q_D = \{P | P \subseteq Q_E\} = 2^{Q_E}$.
3. $q_{0D} = \varepsilon^*(\{q_0\})$
4. $\delta_D(P, a) = \varepsilon^*(\delta_E(P, a))$ ***set extension*** *of* δ_E *and* ***ε-closure***

   where $P \subseteq Q_E$ $(P \in Q_D)$, $a \in \Sigma$, *and*

   $\delta_D(P, a) \subseteq Q_E$ $(\text{or } \delta_D(P, a) \in Q_D)$
5. $F_D = \{F \in Q_D | F \cap F_E \neq \emptyset\}$

Theorem 2.22 *A language L is accepted by some ε-NFA if and only if L is accepted by some DFA.*

Proof*: (If) ε-NFA can* ***easily simulate*** *DFA(DFA* $\Rightarrow$ *ε-NFA).*

(Only if) Eliminating ε-transitions(ε-NFA $\Rightarrow$ *DFA)*

Let $E = (Q_E, \Sigma, \delta_E, q_0, F_E)$ *be a* ε*-NFA and*

DFA $D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$ *is the one in 2.5.5.*

We show $\hat{\delta}_E(q_0, w) = \hat{\delta}_D(q_{0D}, w)$ *by induction on* $|w|$.

basis: $\hat{\delta}_E(q_0, \varepsilon) = \varepsilon^*(q_0)$ *by* <u>*basis*</u> ***definition*** *of* $\hat{\delta}_E$.

$= q_{0D} =$ *by* ***construction*** *of* δ_D.

$= \hat{\delta}_D(q_{0D}, \varepsilon)$ *by* <u>*basis*</u> ***definition*** *of* $\hat{\delta}_D$.

Induction: *Let* $w = xa \in \Sigma^+$, $a \in \Sigma$, *and* $x \in \Sigma^*$.

$\hat{\delta}_E(q_0, xa) = \delta_E(\varepsilon^*(\hat{\delta}_E(q_0, x), a)$ *by* <u>*recursive*</u> ***definition*** *of* $\hat{\delta}_E$.

$= \delta_E(\varepsilon^*(\hat{\delta}_D(q_{0D}, x), a))$ *by* ***induction hypothesis***.

$= \delta_D(\hat{\delta}_D(q_{0D}, x), a)$ *by* ***construction*** *of* δ_D.

$= \hat{\delta}_D(q_{0D}, xa)$ *by* <u>*recursive*</u> ***definition*** *of* $\hat{\delta}_D$.

2.A Extended Finite Automata(XFA)

$X = (Q, \Sigma, q_0, \delta, F)$ *is an* ***extended finite state automata****(XFA), if*

$\delta: Q \times \Sigma^* \rightarrow 2^Q$.

Given an XFA $X = (Q_X, \Sigma, q_0, \delta_X, F_X)$, ***construct***

an ***equivalent*** ε*-NFA* $E = (Q_E, \Sigma, q_0, \delta_E, F_E)$ *such that* $L(X) = L(E)$.

Construction algorithm

$Q_E := Q_X$; $F_E := F_X$; $\delta_E := \varnothing$;

for $^{\forall}(\delta_X(q, x) = P) \in \delta_X$ *where* $x = a_1a_2\ldots a_n$ ***do***

if $0 \le n \le 1 \rightarrow \delta_E(q, x) := P$ *where* $x \in \Sigma \cup \{\varepsilon\}$

$| n \ge 2 \rightarrow \delta_E(q, a_1) = \delta_E(q, a_1) \cup \{p_1\}$;

for $2 \le {}^{\forall}i \le n\text{-}1$ ***do*** $\delta_E(p_{i-1}, a_i) := \{p_i\}$ ***od***; $\delta_E(p_{n-1}, a_n) = P$;

$Q_E := Q_E \cup \{p_1, p_2, \ldots, p_{n-1}\}$ *where* $Q_E \cap \{p_1, p_2, \ldots, p_{n-1}\} \neq \varnothing$

fi

od

The following statements are ***equivalent****,*

1. A language L is ***regular****.*

2. L = L(D) for some DFA D. $\delta : Q \times \Sigma \to Q$.

3. L = L(D) for some DFA D with partial function δ.
$\delta : Q \times \Sigma \to Q \cup \{\emptyset\}$.

4. L = L(N) for some NFA N. $\delta : Q \times \Sigma \to 2^Q$.

5. L = L(E) for some ε*-NFA E.* $\delta : Q \times (\{\varepsilon\} \cup \Sigma) \to 2^Q$.

6. L = L(X) for some XFA X. $\delta : Q \times \Sigma^* \to 2^Q$.