1.A Sets, Relations, Graphs, and Functions

Set *a* ***collection*** *of objects(****element****)*

Let A be a set. If a is the elements in A, we write $a \in A$.

How to specify sets

1. to ***enumerate all*** *of the elements*

2. to state the ***properties*** *that characterizes the elements.*

$A = \{x \mid p(x)\}$

p(x) is a ***predicate***

p(x) is either ***true*** *or* ***false*** *depending on x*

$A = \{x \in U \mid p(x)\}$

$A \subseteq U$, *U is the* ***universe*** *of* ***discourse***

$x \in U$ *U is the* ***type*** *of x in A*

p(x) ***attribute*** *of x*

3. ***automata***, ***grammars***, ***programs***

Three cases for two sets A and B

Case 1. ***subset***

$A \subseteq B$ *or* $B \subseteq A$

$\Leftrightarrow \quad A - B = \varnothing$ *or* $B - A = \varnothing$

$\Leftrightarrow \quad A \cap \overline{B} = \varnothing$ *or* $B \cap \overline{A} = \varnothing$

Case 2. ***disjoint***

$A \cap B = \varnothing$

Case 3. in general(neither subset nor disjoint)

not$(A \subseteq B$ *or* $B \subseteq A)$ *and not* $(A \cap B = \varnothing)$

$\Leftrightarrow \quad A \not\subseteq B$ *and* $B \not\subseteq A$ *and* $A \cap B \neq \varnothing$.

$\Leftrightarrow \quad A \cap \overline{B} \neq \varnothing$ *and* $\overline{A} \cap B \neq \varnothing$ *and* $A \cap B \neq \varnothing$.

Venn diagram

$\overline{A} \cap \overline{B} \stackrel{?}{=} \varnothing$

Cartecian product *of two sets, A and B*

$A \times B = \{(a, b) | \ a \in A, b \in B\}$.

$(a, b) \in A \times B$ ***ordered pair***.

$|A \times B| = |A| \times |B|$.

Binary relation *R from A to B.*

$R \subseteq A \times B$. $a \in A, b \in B, (a, b) \in R$ *or* $a \ R \ b$.

Product *of two relations R and S where* $R \subseteq A \times B$ *and* $S \subseteq B \times C$.

$R \cdot S = \{(a, c) | \ (a, b) \in R, (b, c) \in S\}$

Binary relation R ***on*** *A* $R \subseteq A \times A$.

Identity relation *R on A* $id_A = \{(a, a) | \ a \in A\}$

$^{\forall} R \subseteq A \times A, \ R \cdot id_A = id_A \cdot R = R$

Inverse *of a relation R,* $R^{-1} = \{(b, a) | \ (a, b) \in R\}$

Repeated product *of binary relations.*

Let $R \subseteq A \times A$. *We define*

$R^2 = R \cdot R$, $R^3 = R \cdot R \cdot R$, … $R^n = R \cdot R \cdot \ldots \cdot R$, *and*

$R = R^1$. *Then*

$R^n R^m = R^{n+m}$, *for* $(\forall n, m \in \mathbb{N})$, $n, m \geq 1$.

$R^0 = ?$ *If we define* $R^0 = id_A$,

$R^n R^m = R^{n+m}$, *for* $n, m \geq 0$.

*Another (**recursive**) definition for **repeated product** of binary relations*

$R^0 = id_A$. ***basis***

$R^n = R \cdot R^{n-1}$, $n \geq 1$. ***recursion***

ex) $R^3 = R \cdot R^2 = R \cdot R \cdot R^1 = R \cdot R \cdot R \cdot R^0 = R \cdot R \cdot R \cdot id_A = R \cdot R \cdot R$

Some properties of the binary relations

R is ***reflexive***, *if* $^{\forall}a \in A$, $a\ R\ a$. $\quad id_A \subseteq R$

R is ***symmetric***, *if* $a\ R\ b$ impiles $b\ R\ a$. $\quad R = R^{-1}$

R is ***transitive***, *if* $a\ R\ b$ and $b\ R\ c$ implies $a\ R\ c$. $\quad R \cdot R \subseteq R$

R is ***irreflexive***, *if* $^{\forall}a \in A$, $a\ \not R\ a$. $\quad R \cap id_A = \varnothing$

R is ***asymmetric***, *if* $a\ R\ b$ implies $b\ \not R\ a$. $\quad R \cap R^{-1} = \varnothing$

R *is* ***asymmetric*** $\Rightarrow$ R *is* ***irreflexive***.

R is ***antisymmetric***, *if* $a\ R\ b$ and a $\neq$ b implies $b\ \not R\ a$. $\quad R \cap R^{-1} \subseteq id_A$

R *is* ***asymmetric*** $\Rightarrow$ R *is* ***antisymmetric***.

A ***directed graph*** $G = (V, E)$ *is*

V: *a set of* ***vertices***,

$E \subseteq V \times V$: *a set of* ***edges***,

E: *a* ***binary relation*** *on* V

Let $\mathbb{P}$ = {reflexive, symmetric, transitive} and R' be the $\mathbb{P}$-closure of R.

i) R' is $\mathbb{P}$.

ii) $R \subseteq R'$.

iii) R' is the ***smallest*** *set among satisfying i) and ii).*

$\Leftrightarrow {}^{\forall}$*R" satisfying i) and ii), $R' \subseteq R"$.*

reflexive closure *of R, $R' = R \cup id_A$.*

symmetric closure *of R, $R" = R \cup R^{-1}$.*

transitive closure *of R,*

$R^+ = R^1 \cup R^2 \cup R^3 \cup \ldots = \cup_{i \in N_1} R^i$ *where* $N_1 = \{1, 2, 3, \ldots\}$.

reflexive-transitive closure *of R,*

$R^* = R^0 \cup R^1 \cup R^2 \cup R^3 \cup \ldots = \cup_{i \in N_0} R^i$ *where* $N_0 = \{0, 1, 2, \ldots\}$.

What is the ***reflexive-symmetric-transitive closure*** *of R?*

A binary relation R is ***equivalence****,*

if R is ***reflexive****,* ***symmetric****, and* ***transitive****.*

A binary relation R is ***(ir)reflexive partial order****,*

if R is ***(ir)reflexive****,* ***antisymmetric****, and* ***transitive****.*

Let $R \subseteq A \times A$ *be an* ***equivalence****,*

$[a]_R = \{b \in A | \ a \ R \ b\}$ ***equivalence class****,*

if $a \ R \ b$, $[a]_R = [b]_R$.

$\{[a]_R | \ a \in A\}$ *equivalence* ***partition****.*

$\cup_{a \in A} [a]_R = A$, *if* $a \not R \ b$. $[a]_R \cap [b]_R = \varnothing$.

Power set *of a set A,*

$P(A) = 2^A = \{B | \ B \subseteq A\}$ $B \subseteq A \Leftrightarrow B \in 2^A$.

$|2^A| = 2^{|A|}$.

Let A be a set and ⊕ be a binary operation on A.

$\oplus: A \times A \to A$.

i) $^{\forall}a, b \in A, a \oplus b \in A$. ***closed*** ***algebraic system***

ii) $^{\forall}a, b, c \in A, a \oplus (b \oplus c) = (a \oplus b) \oplus c$ ***associative*** ***semi-group***

binary operation ⇒ n-ary operation

iii) $^{\exists}e \in A$.э. $^{\forall}a \in A, e \oplus a = a \oplus e = a$ ***identity*** ***monoid***

Let (A, ⊕, e) and (B, ⊗, ε) be two monoids.

If *i) h: A → B is a onto function,* $|A| \geq |B|$

ii) h(a⊕b) = h(a)⊗h(b), and *preserve operation*

iii) h(e) = ε. *preserve identity*

Then h is called a ***homomorphism****, and the monoid (B, °, ε) is called a* ***homomrphic*** *to the monoid (A, ⊕, e) w.r.t. h.*

(A, ⊕, e) is called ***concretization*** *of (B, ⊗, ε) and (B, °, ε) is called* ***abstract interpretation*** *of (A, ⊕, e).*

If f is one-to-one and onto, f is called ***isomorphism****.*

Let be $\leq$ *a partial order on A.* $\leq \subseteq A \times A$

Then $(A, \leq)$ *is called as partially ordered set or* ***poset*** *for short.*

Let $(A, \leq)$ *be a poset. We define binary operator on A,*

$\vee, \wedge : A \times A \rightarrow 2^A$

$a \vee b = min \{c \in A | \ a \leq c \text{ and } b \leq c\}$ *least upper bound*

$a \wedge b = max \{c \in A | \ c \leq a \text{ and } c \leq b.$ *greatest lower bound*

If a ***unique*** *lub and a* ***unique*** *glb,*

$\vee, \wedge : A \times A \rightarrow A.$ $(A, \leq)$ *is called as a* ***lattice*** *and*

$(A, \vee, \wedge)$ *is called a* ***algebra*** *induced by the lattice* $(A, \leq)$.

Boolean *algebra,* $(\{f, t\}, \vee, \wedge)$, *is induced by the lattice* $(\{f, t\}, \{f \leq t\})$.

Let A be a sets. Then

Set *algebra on A,* $(2^A, \cup, \cap)$, *is induced by the lattice* $(2^A, \subseteq)$.

Singleton set *algebra,* $(2^{\{a\}}, \cup, \cap)$, *is* ***isomorphic*** *to*

boolean *algebra,* $(\{f, t\}, \vee, \wedge)$ *with respect to* ***bijection*** *g.*

What is the ***bijective*** *function g?*

A binary ***relation*** *from A to B is said to be a* ***function*** *from A to B, if*

1) $^{\forall}a \in A, ^{\exists}(a, b) \in f,$ ***total***

2) $^{\forall}a \in A, ^{\exists_1}(a, b) \in f.$ ***unique***

$f: A \to B$ $(a, b) \in f$ or $a\ f\ b$ or $f(a) = b$ or $f\ a = b.$

Relation *revisited*

i) $R \subseteq A \times B.$ *set of pairs*

ii) $R: A \to 2^B$ *set valued function*

$R(a) = \{b_1, b_2, \ldots, b_n\}$, iff $(a, b_1), (a, b_2), \ldots, (a, b_n) \in R.$

$^{\forall}a \in A, ^{\exists_1}\{b_1, b_2, \ldots, b_n\} \subseteq B$ or $^{\exists_1}\{b_1, b_2, \ldots, b_n\} \in 2^B.$

$\therefore R: A \to 2^B.$

iii) $R: A \times B \to \{$***false, true***$\}$

$R(a, b) =$ ***true***, iff $(a, b) \in R.$

1.B Strings and languages, revisited.

Concatenation of strings, revisited

$\because\ \Sigma^* \times \Sigma^* \to \Sigma^*$.

a ***function from*** *two strings* ***to*** *a string*

a ***binary operation*** *on* ***strings***

(1) $^{\forall}x, y \in \Sigma^*, xy \in \Sigma^*$. ***closed***

(2) $^{\exists}x, y \in \Sigma^*, xy \neq yx$ ***noncommutative***

(3) $^{\forall}x, y, z \in \Sigma^*, x(yz) = (xy)z$ ***associative***

(4) $^{\forall}x \in \Sigma^*, \varepsilon x = x\varepsilon = x$ ε *is the* ***identity*** *element*

$\therefore$ *$(\Sigma^*, \cdot, \varepsilon)$ is a noncommutative* ***monoid****.*

*Another (**recursive**) definition of* Σ^*.

basis	$\varepsilon \in \Sigma^*$ *and* $^\forall a \in \Sigma, a \in \Sigma^*$.
recursion 1	*If* $x, y \in \Sigma^*$, *then* $xy \in \Sigma^*$.
recursion 2	*If* $x \in \Sigma^*$ *and* $a \in \Sigma$, *then* $xa \in \Sigma^*$.

Extend *the domain and range of the concatenation from* ***strings*** *to* ***languages(set of strings)***

$\cdot : 2^{\Sigma^*} \times 2^{\Sigma^*} \to 2^{\Sigma^*}$.

Let $L, S \subseteq \Sigma^*$ *(or* $L, S \in 2^{\Sigma^*}$*). Then we define*

$$LS = \{xy \mid x \in L, y \in S\}$$
$$|LS| \le |L| \times |S|$$

$(2^{\Sigma^*}, \cdot, \{\varepsilon\})$ *is a (induced) noncommutative* ***monoid***.

power of an alphabet revisited

$\Sigma^0 = \{\varepsilon\}$ **basis**

$\Sigma^n = \Sigma\Sigma^{n-1}$ *for* $n \geq 1$ **recursion**

$|\Sigma^n| = |\Sigma|^n$.

We can ***extend*** *to the power of* ***languages***

$L^0 = \{\varepsilon\}$ **basis**

$L^n = LL^{n-1}$ *for* $n \geq 1$ **recursion**

$|L^n| \leq |L|^n$.

$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots = \cup_{i \in N_0} L^i$.

$L^+ = L^1 \cup L^2 \cup L^3 \cup \ldots = \cup_{i \in N_1} L^i$.

For any $L \subseteq \Sigma^*$, $L^* \subseteq \Sigma^*$.

$\varepsilon \notin \Sigma^+$, *but* $\varepsilon \in L^+$, *only if* $\varepsilon \in L$.

String, revisited

Let x be a string over Σ. *Then we write* $x \in \Sigma^*$.
Let $|x| = n(n \geq 0)$. *Then*

$x = a_1a_2 \ldots a_n \in \Sigma^n$ *or* Σ^* *(if* $n=0$, $x = a_1a_2 \ldots a_n = \varepsilon$*)*

where $1 \leq^{\forall} i \leq n$, $a_i \in \Sigma$.

Consider a function $x: \{1, 2, \ldots, n\} \rightarrow \Sigma$.

$1 \leq^{\forall} i \leq n$, *if* $x(i) = a_i$, *we can write* $x = (a_1, a_2, \ldots, a_n)$

$a_1a_2 \ldots a_n \leftrightarrow^{1:1} (a_1, a_2, \ldots, a_n)$

The ***strings*** *over* Σ *of length* $n(\Sigma^n)$,

is ***isomorphic*** *to the* ***functions*** *from* $\{1, 2, \ldots, n\}$ *to* Σ *w.r.t. f.*

Let $B^A = \{f | f: A \rightarrow B\}$. *Then* $|B^A| = |B|^{|A|}$.

$|\Sigma^n| = |\Sigma^{\{1, 2, \ldots, n\}}| = |\Sigma|^n$.

Let $x = a_1a_2 \ldots a_n$ be a string of length n and $k \geq 0$.

$x^R = a_n a_{n-1} \ldots a_2 a_1$. ***reversal** of x.*

$k{:}x = a_1a_2 \ldots a_k$, *if* $k \leq n$; ***prefix** of x with length k.*

$= x$, *otherwise.*

$x{:}k = (k{:}x^R)^R$. ***suffix** of x with length k.*

1.C Set isomorphism and infinite sets

If there exists a ***bijection****(짝짓기 ,* ***onto 1-1****) f from A to B,*

two sets A and B have same ***cardinality****, written* $|A| = |B|$*, and*
two sets A and B are said to be ***isomorphic*** *w.r.t. f,* $A \cong_f B$.

function f is ***onto***(***surjection****;* ***correspondence***), *if*

$\forall b \in B, \exists a \in A$.ə. $f(a) = b$.

$|A| \geq |B|$

f^{-1} *is* ***total*** *but* ***not*** *unique function.*

function f is ***one-to-one***(***injection***), if

$\forall a_1, a_2 \in A$, $a_1 \neq a_2$ implies $f(a_1) \neq f(a_2)$.

if $\exists b \in B$.ə. $f(a) = b$, $\exists_1 a \in A$.

$|A| \leq |B|$

f^{-1} *is* ***unique*** *but* ***not*** *total function.*

function f is ***one-to-one*** *and* ***onto*****(*****bijection*****).***

$^{\forall}b \in B, {}^{\exists}a \in A$.ə. $f(a) = b. \wedge {}^{\forall}a_1, a_2 \in A, a_1 \neq a_2$ implies $f(a_1) \neq f(a_2)$.

$^{\forall}b \in B$, *if* $^{\exists_1}a \in A$.ə. $f(a) = b$.

$|A| = |B|$

f^{-1} *is a* ***function****.*

Let f be a bijective(1-1 and onto) function from A to B.

$f: A \leftrightarrow B$.

f is a function from A to B, and

f^{-1} *is a function from B to A.*

$^{\forall}a \in A, {}^{\exists_1}f(a) \in B \wedge {}^{\forall}b \in B, {}^{\exists_1}f^{-1}(b) \in A$.

The set A and B are ***isomorphic*** *w.r.t. f,* $A \cong_f B$.

Two sets A and B has same ***cardinality****, if there is a bijective function between them.*

A set is said to be ***countable*** *(****enumerable,*** *(****finite*** *or* ***infinite****)),*
if it has the ***same cardinality*** *with a* ***subset*** *of* $\mathbb{N}$*,*
and ***uncountable*** *(****infinite****), otherwise.*

A set is ***countably infinite****, if it has the* ***same cardinality*** *with* $\mathbb{N}$*.*
the ***cardinality*** *of* $\mathbb{N}$ *is denoted as* $\aleph$*,* $|\mathbb{N}| = \aleph$*.*

Let A be ***countable****. Then we can* ***enumerate*** *the set in* ***numeric*** *order.*

$A = \{a_0, a_1, \ldots, a_n\}$ ***finite*** *for some* $n \geq 0$*.*

$A = \{a_0, a_1, \ldots\}$ ***infinite(countable****,* ***enumerable****)*

Consider

$N_1 = \{1, 2, 3, \ldots\}$ $\quad |N_1| = |\mathbb{N}| = \aleph$, *but* $N_1 \subset \mathbb{N}$.

$E = \{e \in \mathbb{N} \mid e = 2i, i \in \mathbb{N}\}$ $\quad |E| = |\mathbb{N}| = \aleph$, *but* $E \subset \mathbb{N}$.

$I = \mathbb{N} \cup \{-i \mid i \in \mathbb{N}\}$ $\quad |I| = |\mathbb{N}| = \aleph$, *but* $\mathbb{N} \subset I$.

$Q = \mathbb{N} \times \mathbb{N}$ $\quad |Q| = |\mathbb{N}| = \aleph$.

enumerate $(i, j) \in \mathbb{N} \times \mathbb{N}$ *in* (***natural) numeric*** *order*

$\mathbb{N} \times \mathbb{N} = \{\underline{(0, 0)}, \underline{(1, 0), (0, 1)}, \underline{(2, 0), (1, 1), (0, 2)}, \underline{(3, 0), (2, 1)}, \ldots\}$

$f(i, j) = 1 + 2 + 3 + \ldots + (i+j) + j$ $\qquad f(0, 0) = 0$

$= (i+j)(i+j+1)/2 + j$

$f: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ *is one-to-one and onto*

$\therefore |\mathbb{N} \times \mathbb{N}| = |\mathbb{N}| = \aleph$.

Prove $|\Sigma^*| = \aleph$.

Proof Canonical(lexicographical) order *for* Σ^*.

in order of ***size*** *and if same size,* ***alphabetic order****.*

Let $|\Sigma| = k$. *Then we can alphabetic order* $\Sigma = \{a_0, a_1 \ldots, a_{k-1}\}$,

and we can order $x \in \Sigma^*$ *where* $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots = \cup_{i \in N_0} \Sigma^i$.

$\Sigma^* = \{\varepsilon, a_0, a_1 \ldots, a_{k-1}, a_0a_0, a_0a_1, \ldots, a_0a_{k-1}, a_1a_0, \ldots, a_{k-1}a_{k-1}, a_0a_0a_0, \ldots, a_{k-1}a_{k-1}a_{k-1}, \ldots\}$

If $x = a_1a_2 \ldots a_n$, *the order of x, f(x), is n-digit k-ary number plus base.*

$$f(x) = k^0 + k^1 \ldots + k^{n-1} + a_1k^{n-1} + a_2k^{n-2} + \ldots + a_nk^0$$

$$= (k^n - 1)/(k-1) + a_1k^{n-1} + a_2k^{n-2} + \ldots + a_nk^0.$$

We ***enumerate*** $x = a_1a_2 \ldots a_n \in \Sigma^*$ *in* ***numeric order***

$\therefore$ $f:\Sigma^* \rightarrow \mathbb{N}$ *is one-to-one onto. Q.E.D.*

Consider $\{0, 1\}^{\mathbb{N}}$*:* ***infinite*** *binary strings (See pp.12)*

and $2^{\mathbb{N}}$*:* ***power*** *set of natural numbers (Note that* $2^A = \{B | B \subseteq A\}$

Cantor's diagonal argument

Assume $2^{\mathbb{N}}$ *is* ***countable****.*

We can ***enumerate*** $|2^{\mathbb{N}}|$ *subsets of* $\mathbb{N}$*, in numeric order as follows,*

$2^{\mathbb{N}} = \{a_0, a_1, \ldots, a_i, \ldots\}$ *where*

$\forall i \in \mathbb{N}, a_i \leftrightarrow (b_{i0}, b_{i1}, \ldots b_{ii}, \ldots) \in 2^{\mathbb{N}}$

$\forall j \in \mathbb{N}$, *if* $b_{ij} = 1$ *then* $j \in a_i \in 2^{\mathbb{N}}$,

if $b_{ij} = 0$ *then* $j \notin a_i \in 2^{\mathbb{N}}$.

$\therefore$ ***Power set*** *of* $\mathbb{N} \leftrightarrow$ ***infinite binary string***

Power set of integers and infinite binary strings are ***isomorphic****.*

$\therefore |2^{\mathbb{N}}| = |\{0, 1\}^{\mathbb{N}}|$.

Consider $a \leftrightarrow (\bar{b}_0, \bar{b}_1, \ldots \bar{b}_i, \ldots)$ *where* $^{\forall}i \in \mathbb{N}$, $\bar{b}_i = 1$, *if* $b_{ii} = 0$,
$\bar{b}_i = 0$, *if* $b_{ii} = 1$.

Since $^{\forall}i \in \mathbb{N}$, $\bar{b}_i \neq b_{ii}$, $\therefore$ $^{\forall}i \in \mathbb{N}$, $a \neq a_i$.

$\therefore a \notin \{a_0, a_1, \ldots, a_i, \ldots\}$.

But $a \in 2^{\mathbb{N}}$ *by the* ***definition*** *of power set.*

$a \notin \{a_0, a_1, \ldots, a_i, \ldots\}$ *but* $a \in 2^{\mathbb{N}}$.

$\therefore$ *Contradiction!!!*

We ***fail*** *to* ***enumerate*** $2^{\mathbb{N}} = \{a_0, a_1, \ldots, a_i, \ldots\}$ *in* ***numeric order****!*

$\therefore$ *We* ***conclude*** *that* $|2^{\mathbb{N}}| > |\{a_0, a_1, \ldots, a_i, \ldots\}| = \aleph$.

$2^{\mathbb{N}}$ *is* ***uncountable****.*

$\{0, 1\}^*$ *vs* $\{0, 1\}^{\mathbb{N}}$.

$\{0, 1\}^*$: ***all(countably*** *infinite union of* ***finite*** *binary strings*

$= \{0, 1\}^0 \cup \{0, 1\}^1 \cup \{0, 1\}^2 \cup \{0, 1\}^3 \cup \ldots$

$= \{\varepsilon\} \cup \{0, 1\} \cup \{00, 01, 10, 11\} \cup \{000, \ldots, 111\} \cup \ldots$

$= \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, \ldots, 111, \ldots\}$

$\{0, 1\}^{\mathbb{N}}$: ***all(uncountaby*** *infinite union of)* ***infinite*** *binary strings*

$= \{000\ldots000\ldots,$ $\leftrightarrow \{\}$

$100\ldots000\ldots,$ $\leftrightarrow \{0\}$

$010\ldots000\ldots,$ $\leftrightarrow \{1\}$

$110\ldots000\ldots,$ $\leftrightarrow \{0, 1\}$

$\ldots,$

$111\ldots111\ldots\}$ $\leftrightarrow \{0, 1, 2, \ldots\} = \mathbb{N}$

$|\{0, 1\}^{\mathbb{N}}| = |2^{\mathbb{N}}| > |\{0, 1\}^*| = \aleph.$

Cantor's diagonal argument

Complement of diagonal element

Russel's paradox

$S = \{x \mid x \notin x\}$

$x \in S$, iff $x \notin x$. *But $S \in S$, iff $S \notin S$.* ***contradictory!***

Halting problem

H(P): ***if*** *halt(P, P) then loop forever*

elses *not halt(P, P) then stop* ***fi***

What happens if H(H) stops or loops forever?

Denial of self recursion

Σ^* *is* ***countable****.* ***strings*** *are* ***countable***

But is 2^{Σ^*} ***uncountable****.* ***languages*** *are* ***uncountable***

class of languages

N. Chomsky

Finite(countable)

Countably infinite

natural numbers, integers, rational numbers,

***finite** strings …*

Uncountable

Cantor's diagonal argument

***power set** of natural numbers*

***infinite** strings*

real numbers

*Some informal descriptions on **countable** and **uncountable** infiniteness*

$\aleph \pm k = \aleph$	$\aleph \times k = \aleph$	***countable***
$\aleph \times \aleph = \aleph$	$\aleph^k = \aleph$	***countable***
But $k \times k \times \ldots = k^{\aleph} > \aleph$		***uncountable**($k \geq 2$)*