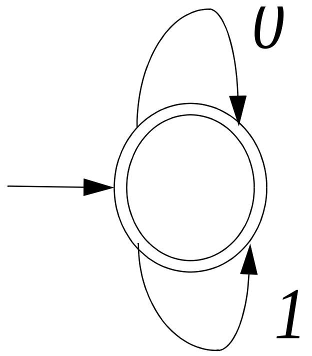
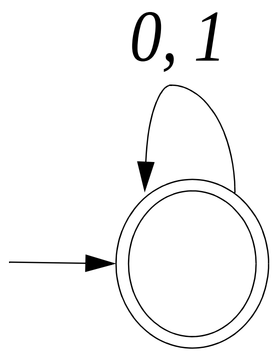
2.B Examples of DFA's Du and Ko's 부교재 Example 2.6-2.15(p28-37).

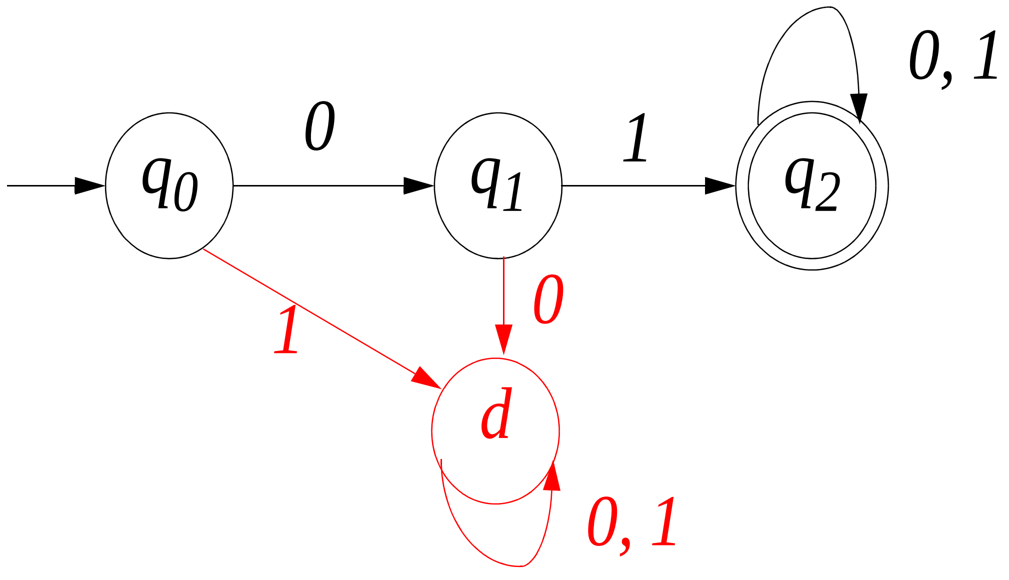
Example 2.6





$(0+1)^*$.

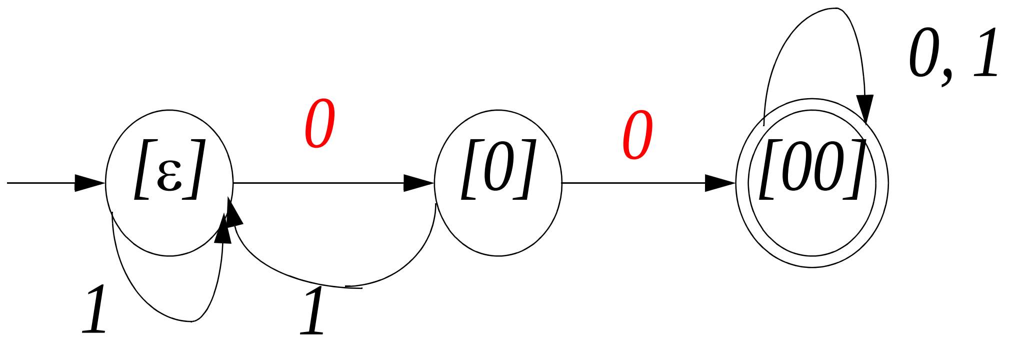
Example 2.7 *The set of all binary strings beginning with* ***prefix*** *01.*



$01(0+1)^*$.

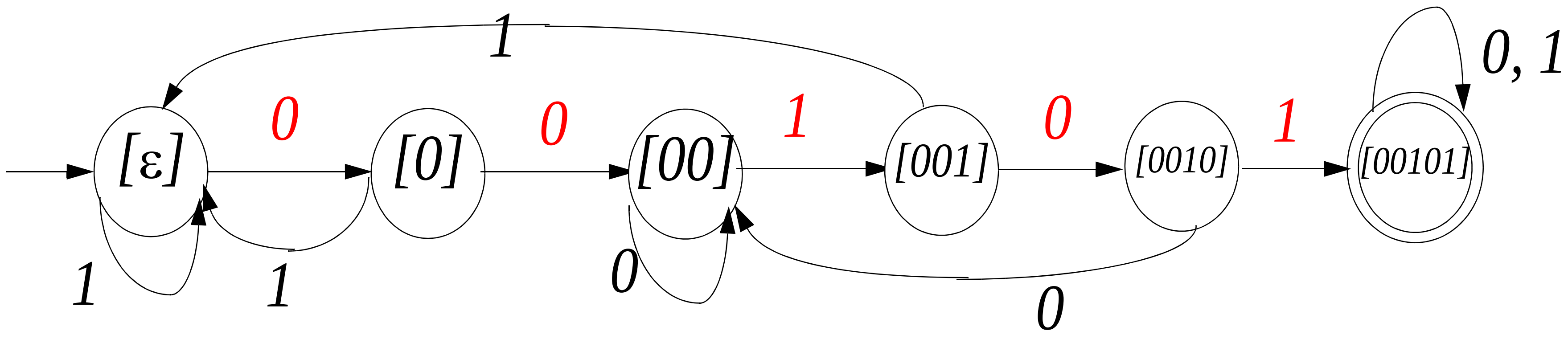
Example 2.8 *The set of all binary strings having a* ***substring*** *00.*

$$(0+1)^{*}00(0+1)^{*}$$



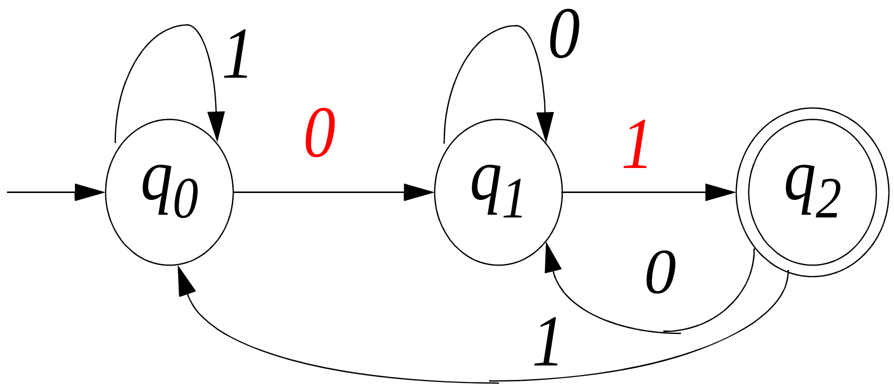
Example 2.9 *The set of all binary strings having a* ***substring*** *00101.*

$$(0+1)^{*}00101(0+1)^{*}.$$



Example 2.10 *The set of all binary strings ending with* ***postfix*** *01.*

$(0+1)^*01.$



1. Strings ***begining*** *with* $a_1 \ldots a_k$. *(****prefix*** $a_1 \ldots a_k$*)* *r.e.* $a_1 \ldots a_k T^*$.

$M = (\{q_0, \ldots, q_k, d\}, T, \delta, q_0, \{q_k\})$ *(k+2) states:* $q_0, q_1, \ldots, q_k$ *and* d.

$0 \le {}^\forall i < k$:

q_i *stands for "the prefix* $a_1 \ldots a_i$ *of the* ***prefix*** $a_1 \ldots a_k$ *is* ***found****."(****i*** $\le k$*)*

$\delta(q_i, a_{i+1}) = q_{i+1}$,

${}^\forall \mathbf{a} \in \Sigma - \{a_{i+1}\}: \delta(q_i, \mathbf{a}) = d$ ***fail*** *to get prefix* $a_1 \ldots a_k$.

d *stands for "****fail*** *to get prefix* $a_1 \ldots a_k$*."*

${}^\forall a \in \Sigma, \delta(d, a) = d$. *dead state*

q_k *stands for "****prefix*** $a_1 \ldots a_k$ *is already* ***found****."* *final state*

${}^\forall a \in \Sigma, \delta(q_k, a) = q_k$.

2. *Strings having a* ***substring*** $a_1\ldots a_k$. (***substring*** $a_1\ldots a_k$) r.e $T^*a_1\ldots a_kT^*$.

$M = (\{q_0, \ldots, q_k\}, T, \delta, q_0, \{q_k\})$ $(k+1)$ states: $q_0, q_1, \ldots, q_k$.

$0 \le {}^\forall i < k$:

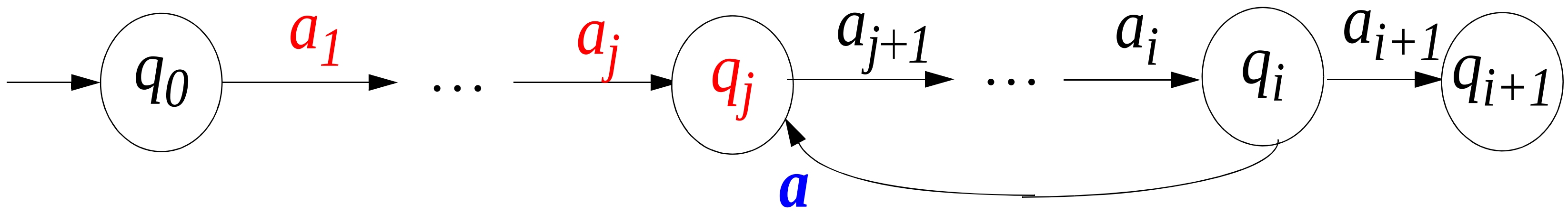
q_i stands for "the prefix $a_1\ldots a_i$ of the **substring** $a_1\ldots a_k$ is **found**."($i \le k$)

$$\delta(q_i, a_{i+1}) = q_{i+1},$$

$${}^\forall a \in \Sigma - \{a_{i+1}\}: \delta(q_i, a) = q_j.$$

where $j (0 \le j \le i)$ is the **maximum** index .϶. $a_1\ldots a_j = a_{i-j+2}\ldots a_{i-1}a_i a$. $(a_j = a)$

Note that $|a_{i-j+2}\ldots a_{i-1}a_i a| = i-(i-j+2)+1+1 = j$.



q_k stands for "**substring** $a_1\ldots a_k$ is already **found.**" final state

$${}^\forall a \in \Sigma, \delta(q_k, a) = q_k.$$

*3. Strings **ending** with $a_1 \ldots a_k$. (**suffix** $a_1 \ldots a_k$)* *r.e.* $T^* a_1 \ldots a_k$.

$M = (\{q_0, \ldots, q_k\}, T, \delta, q_0, \{q_k\})$ *(k+1) states:* $q_0, q_1, \ldots, q_k$.

$0 \le \forall i \le k$:

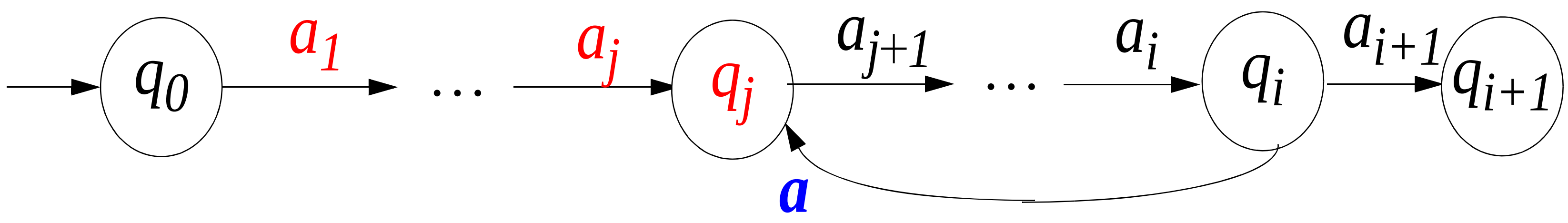
q_i *stands for "the prefix* $a_1 \ldots a_i$ *of the **suffix*** $a_1 \ldots a_k$ *is **found**."*($i \le k$)

$\delta(q_i, a_{i+1}) = q_{i+1}$;

$\forall \mathbf{a} \in \Sigma - \{a_{i+1}\}$: $\delta(q_i, \mathbf{a}) = q_j$, *if* $\mathbf{a} \ne a_{i+1}$*(otherwise).*

where $j(\le i)$ *is the **maximum** index .϶.* $a_1 \ldots a_j = a_{i-j+2} \ldots a_{i-1} a_i \mathbf{a}$.

Note that $|a_{i-j+2} \ldots a_{i-1} a_i \mathbf{a}| = j$.



In the final state q_k,

a suffix $a_1 \ldots a_k$ *is found but search for the next **suffix**, if any,*

Example 2.11 *The set of binary expressions of positive integer which are congruent to m modulo $k (0 \le m < k)$.*

Consider k-states $q_0, \ldots, q_{k-1}$,

q_i *stands for "$y \equiv i \pmod{k}$", $0 \le i < k$.*

basis: $\delta(q_0, \varepsilon) = q_0$ and $\varepsilon \equiv 0 \pmod{k}$.

rec: Suppose $\delta(q_0, x) = q_i$ and $\delta(q_0, xa) = \delta(\delta(q_0, x), a) = \delta(q_i, a) = q_j$.

$j \equiv xa \pmod{k} \equiv 2i + a \pmod{k}$ for $a \in \{0, 1\}$

Final states = $\{q_m\}$

Special case m=0 and no leading zeros

new start state q_0' and dead state d,

$\delta(q_0', 0) = d$, $\delta(q_0', 1) = q_{2*0+1 \pmod{k}} = q_{1 \pmod{k}} = q_1$.

$\delta(d, 0) = \delta(d, 1) = d$.

If no leading zeros except 0.

$\delta(q_0', 0) = f$, $\delta(q_0', 1) = q_1$, $\delta(f, 0) = \delta(f, 1) = d$, $\delta(d, 0) = \delta(d, 1) = d$.

Example 2.15 *The set of binary strung in which every block of four consequtive symbols contains a substring 01.*

Consider the complemet $\overline{L}$ contains a substring 0000, 1000, 1100, 1110 or 1111.