제 24강 - primitive recursive function (11/30 木)

3. Primitive Recursion of functions: f

ex) $f : \mathbb{N}^2 \to \mathbb{N}$ s.t $f(x,y)$ = # of nodes in a full x-nary balanced tree whose depth is y

1) level n has x^n nodes
2) depth $y+1$: add $x^{y+1} = x^y \cdot x$ nodes to $f(x,y)$: depth y

In general,

$$f(x,0) = g(x)$$
$$f(x,y+1) = h(x,y,f(x,y))$$

or

$$f(\bar{x}, \sigma(y)) = g(\bar{x})$$
$$f(\bar{x}, \sigma(y)) = h(\bar{x}, y, f(\bar{x},y))$$

$$f : \mathbb{N}^{k+1} \to \mathbb{N}^m$$
$$g : \mathbb{N}^k \to \mathbb{N}^m$$
$$h : \mathbb{N}^{k+m+1} \to \mathbb{N}^m$$

$\bar{x} \in \mathbb{N}^k,\ y \in \mathbb{N}$

Constant fcts

$K_m^n : \mathbb{N}^n \to \{m\}$, $m, n \in \mathbb{N}$

where $K_m^n(\bar{x}) = m$, $\bar{x} \in \mathbb{N}^n$

$$expo(x,0) = K_1^1(x)$$
$$expo(x,\sigma(y)) = (\pi_1^3 \times \pi_3^3) \circ mult(x,y,expo(x,y))$$

$$expo(x,0) = 1$$
$$expo(x,y+1) = mult(x, expo(x,y))$$

Predecessor fct

$Pred : \mathbb{N} \times \mathbb{N}$

$Pred(0) = \xi() \quad = 0$

$Pred(\sigma(x)) = \pi_1^2(x, pred(x)) \quad = x$

Ackermann function

$A : \mathbb{N}^2 \to \mathbb{N}$

$A(0,y) = y+1$

$A(x+1,0) = A(x,1)$

$A(x+1,y+1) = A(x, A(x+1,y))$