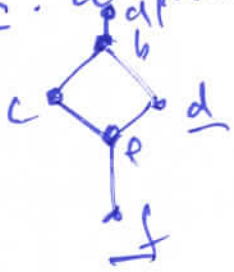
5/12 (7)(9) Boolean algebra

Lattice $(A, \leq)$

$\leq$: a partial order A: poset partially ordered set



$ub(\leq d, f) = \{a, b, d\}$

$lub(d, f) = \{d\}$.

unique lub, glb.

(V) $lub: A \times A \to A$ > operation

($\wedge$) $glb: A \times A \to A$

algebraic system.

$(A, \vee, \wedge)$ -- algebraic system

poset (lattice) $\xrightarrow{\text{define}}$ an algebraic system

A Lattice $(P(S), \subseteq) \xrightarrow[\text{define}]{\vee, \wedge} (A, \cup, \cap)$

$\forall A, B \in P(S) = 2^S = \{A \subseteq S\}$

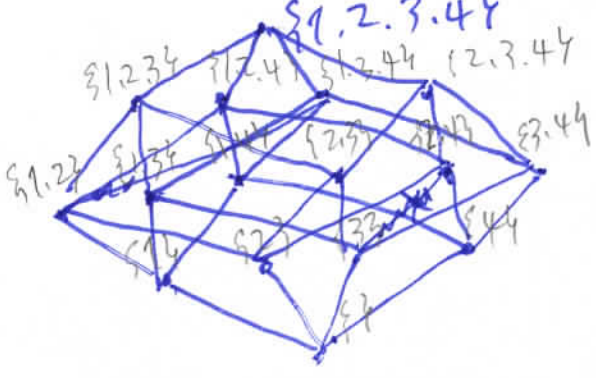
$A, B \subseteq S$ $A \vee B = C$ $C \supseteq A, B$

$S = \{1.2.3.4\}$ $A = \{1.2.\}$, $B = \{2, 3\}$

$A \vee B = \{1.2.3\} = A \cup B$ $B' = \{3.4\}$

$A \wedge B = \{2\} = A \cap B$

$A \wedge B' = \{\}$



Cantor's Diagonal Arg (Chap 2)

$f: \mathbb{N} \to \{0,1\}$

$\updownarrow$

~~$A \subseteq \mathbb{N}$~~

$2^{\mathbb{N}}$

2) $(\mathbb{N}, \leq)$ lattice

$1 \vee 3 = 3, = \max(1,3)$

$1 \wedge 3 = 1 = \min(1,3)$

$(\mathbb{N}, \geq)$ 12

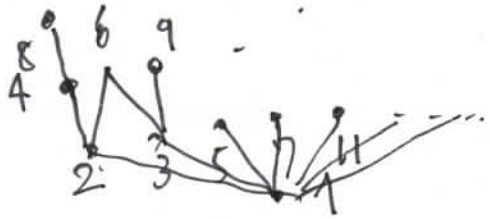
$(\mathbb{N}, \min, \max)$

$(\mathbb{N}, \max, \min)$ algebraic system!

$(\mathbb{N}, |)$ lattice $a|b$ if a is a multiple of b

$\vee(2,6) = 6$ — lcm (최소공배수)

$\wedge(2,6) = 2$ — gcd (최대공약수)



prime numbers

$a \times (b+c) = (a \times b) + (b \times c)$

$a + (b \times c) \neq (a+b) \times (a+c)$

fin. te

boolean lattice

Powerset algebra $(2^S, \cup, \cap, -)$