1) 5/7(Fri) Algebraic System 1

↳ Chap 11 & 12 of Liu's book.

① Alg. system

$(A, \oplus)$ A : a set (finite or nonfinite) ("infinite")

$\oplus$: a $\boxed{\text{binary}}$ operation on A. (연산)

$\oplus : A \times A \to A$

※ function vs operation

$f : A \to B$

$\oplus : A \to A$

$A \times A \to A$

⋮

$A \times A \times \cdots A \to A$

Type ~~=~~ is a set

$2^{31} = (2^{10})^3 \cdot 2 = (K)^3 \cdot 2 = 2G$

$(\mathbb{N}, +)$ $\forall a, b \in \mathbb{N}$ $\exists_1 c \in \mathbb{N}$ $(+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N})$

+ : closed (닫혀있다) ... $\boxed{\mathbb{N} \text{ is infinite}}$

$3 \in \mathbb{N}, 5 \in \mathbb{N}$ $3+5 \in \mathbb{N}$ ($3+5 = 8$)

Integer in programs : 32 bit integer : $-2^{31} \sim 2^{31} - 1$ (2's complement)

addition on Integers on P.L's are not closed!

infix $a \oplus b$ ... ambiguity! $a \oplus b \oplus c$

prefix $\oplus a b$ ↳ 애매모호함. $= (a+b) \oplus c$ or $= a \oplus (b \oplus c)$

postfix $a b \oplus$ 중의성. 헷갈림 (!)

⋮

↓

$f(a_1, \cdots, a_n)$

※ 3-operator

<e> ? <e> : <e> in C

IF

② semigroup $(A, \oplus)$

i) $(A, \oplus)$ is an $\boxed{\text{n-ary}}$ algebraic system (binary)

ii) $\oplus$ is associative

$\forall a, b, c \in A, \ (a \oplus b) \oplus c = a \oplus (b \oplus c)$

Pascal $\sum_{i=1}^{10} i =$

$\sum_{i=1}^{n} i^2 \ \cancel{f}((1^2 + 2^2) + 3^2) + \cdots + n^2) = \sum_{i \in \{1, \cdots, n\}} i^2$

3) String over an alphabet Σ : $x \in \Sigma^*$

$\{1, 2, 3, \dots\}$

$\{1, 2, 3, \dots, n\}$

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \cdots$$
$$= \sum_{i \in \mathbb{N}} \Sigma^i$$

(※ Σ^i $\quad a_1, a_2, \dots, a_i \in \Sigma$
$\quad a_1 a_2 \cdots a_i \in \Sigma^i$)

① $\Sigma^0 = \{\varepsilon\}$ where $\varepsilon \cdot x = x \cdot \varepsilon = x \ \forall x \in \Sigma^*$

$\Sigma^i = \Sigma \cdot \Sigma^{i-1} \ (i \geq 1)$

Ex) $\Sigma = \{a, \dots, z\}$ $\quad \Sigma^* = \{\varepsilon, a, b, c, \dots, z, aa, ab, \dots, zz$

한글 $\Sigma = \{$ㄱ ... ㅎ, ㅏ, ... ㅣ$\}$ — 24字

ㄱ ㅏ ㄱ ㄱ ㄱ ㅗ < 갂고 / 각꼬

$\Sigma = \{$ㄱ ... ㅎ, ㄲ, ㄸ, ㅃ, ㅆ, ㅉ, ㅏ, ... ㅣ$\}$ 29字

Σ^* : Universe of ~~Kor~~ 한글

(초·중·종)* where $\varepsilon \in$ 종성.

$19 \times 21 \times 28 = 1,1xxx$ ... superset of 한글

$(\Sigma^*, \cdot)$ is a semigroup

$(x \cdot y) \cdot z = x \cdot (y \cdot z)$

~~Let e be a identity~~

③ Let $(A, \oplus)$ be a semigroup.

Let $e \in A$ be an identity if $\forall a \in A$, $e \oplus a = a \oplus e = a$.

Then $(A, \oplus, e)$ is a monoid.

④ Let $(A, \oplus, e)$ be a monoid.

$a \in A$ and $a^{-1} \in A$, if $a \oplus a^{-1} = e$ Then a^{-1}: inverse of a.

If $\forall a \in A$, $\exists a^{-1} \in A$. Then

$(A, \oplus, e)$ is a group.

Ex) $(\mathbb{Z}, +, 0)$ ... is a group!

↑ integer

$(\mathbb{Q}, *, 1)$ ... " ? 0^{-1}: X

↑ rational number

Example in TP page $(Z_n, \oplus)$

⊕	홀	짝
홀	짝	홀
짝	홀	짝

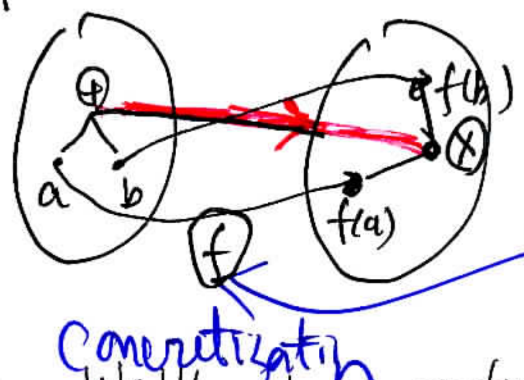
⟺

	O	E
O		
E		

Homomorphism: ① $f: A \to B$.

$(A, \oplus)$ and $(B, \otimes)$ are algebraic systems.

② $f(a \oplus b) = f(a) \otimes f(b)$, $\forall a, b \in A$, $f(a) \in B$, $f(b) \in B$



2+3 = 5

→ 짝⊕홀 = 홀

abstraction

Concretization

월요일 사진 찍는다 — We'll take picture on Mon!