1)

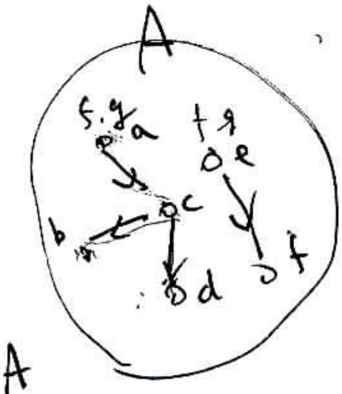
4/28 (Wed) Graphs

Let A (and C) be a set. $R \subseteq A \times A$, $f: A \to 2^C$

. relation

. graph

g



$\underline{A, R, g}$: input

f: output

$\forall a \in A$

$f(a) \supseteq g(a)$

$f(a) \supseteq f(b)$ ∃. aRb.

↑ bounded var

↳ free variable
unbounded variable
(undefined var. in PL.)

→ not you fix (bound, define) b

If $\boxed{f(a) = g(a) \cup \bigcup_{aRb} f(b)}$ Then $f(a) \supseteq g(a) \cup f(c)$

$g(c) \cup f(b) \cup f(d)$ $= g(a) \cup g(c) \cup (g(b) \cup g(d))$

$g(b) \cup \{\}$ $g(d) \cup \{\}$

$f(a) = \bigcup_{aR^*b} g(b)$ $= g^*(a)$

$R(a) = \{ \cdots \} = \{c\}$, $R(c) = \{b, d\} \ldots$

$R^*(a) = \{ \cdots \} = \{a, c, b, d\}$

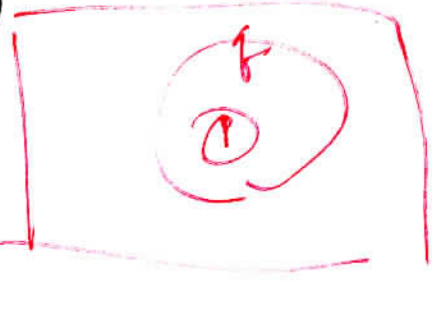
Equivalence. rela ($R \subseteq A \times A$) ⟺ partition — exhaustive, disjoint

i) ref. $\forall a \in A$ aRa (or $(a,a) \in R$, $R(a) \ni \{a\}$) $a \in R(a)$

ii) sym. $\forall a, b \in A$ $aRb \Rightarrow bRa$

if aRb then bRa.

iii) tran $\forall a, b, c \in A$ $aRb \land bRc \Rightarrow aRc$.



1

Relation: Three faces

① $R \subseteq A \times B$

$(a, b) \in R$ where $a \in A$, $b \in B$

② $R: A \times B \to \{t, f\}$

: a binary relation from A to B.

aRb where $a \in A$, $b \in B$

Ex: $3 \le 5$. $\le: N \times N \to \mathbb{B}$ ($= \{t, f\}$)

③ $R: A \to 2^B$.

a set valued ftn

$R(a) = \{b_1, b_2, \ldots, b_n\}$

where $a \in A$

$b_1, \ldots b_n \in B$

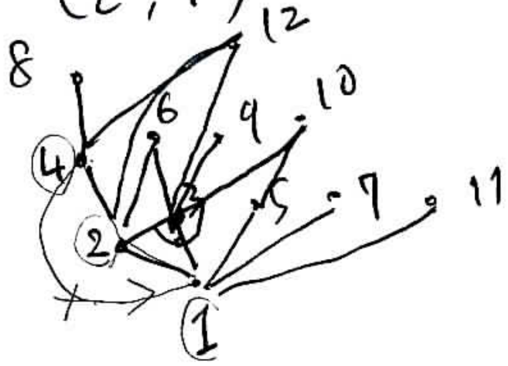
($\{b_1 \cdots b_n\} \in 2^B$ or $\subseteq B$)

另外

poset

$(Z^+, |)$

Hesse diagram



$(2^S, \subseteq)$ assume $\underline{S=\{1,2,3\}}$ 2^3 subsets