1

Recurrence relation 2.

A linear rec. relation of degree k $\longleftrightarrow$ a polynomial eq of ~~order~~ degree k.

Ex. 3.5 $h_n = 2h_{n-1}$ vs $H_n = 2H_{n-1} + 1$ (TBD)

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$h_n = 2^{n-1}$ $\qquad\qquad$ $H_n = 2^n - 1$

$f_2 = \frac{1}{\sqrt{5}}\left(\frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}\right)$

$= \frac{1}{\sqrt{5}} \cdot \frac{4\sqrt{5}}{4} = 1$

$f_3 = \cdots$

$f_4 = \cdots$

Ex. 4 Fib. number

~~typo~~ try $f_0, f_1, f_2, f_3, f_4, \ldots$

~~typo~~ try f_0, f_1 and $f_{n-1} + f_{n-2} \Rightarrow f_n$.

Thm 2. Double root.

C.E. $(r - r_0)^2 = 0$.

$\frac{d}{dr}(\quad) = 2(r - r_0) = 0$.

$a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$ mul. 2 (n) $\rightarrow$ proof is left to students.

HW #7. 6

Assume

Ex. 5 $a_n = 6a_{n-1} + 9a_{n-1}$ typo. $\to$ 2

Thm 3. degree 2 $\Rightarrow$ degree k (distinct roots)

Thm 4. degree k. (multiple roots)

$\alpha_1 r_0^n + \alpha_2 n r_0^n + \alpha_3 n^2 r_0^n$

mult. 3 root r_0 of. degree k

t roots $r_1, r_2, \ldots r_t$ $(t \le k)$

long eg. in TP#8. mult. m_1 m_2 $\ldots$ m_t $\sum^t m_i = k$

Ex 7. 2, 2, 2. 5, 5. 9.

$\underbrace{2,2,2}_{3}\ \underbrace{5,5}_{2}\ \underbrace{9}_{1}$

※ typo in page 8

$(\ \ )3^n$

⇓

$(\ \ )5^n$

$(a+bn+cn^2)2^n + (d+en)5^n + (f)9^n$

2nd order — mult. 3

1st order — mult. 2

0th order — mult. 1

Nonhomogeneous $P(m)=0$ — homoge..

$P(m)=F(n)$ — non-homoe.

Let $a_n^{(h)}$ be a solution of $\underbrace{P(m)=0}_{\text{homog. r.r.}}$.

$$a_n = a_n^{(h)} + a_n^{(p)}$$

Ex. 10 $\underline{a_n = 3a_{n-1} + 2n}$ with $a_1 = 3$.

(H). $\underline{a_n - 3a_{n-1} = 0}$

$r=3.\quad a_n^{(h)} = \alpha\cdot 3^n$

(P) $F(n) = 2n. \rightarrow \underline{a_n^{(p)}} = \cancel{an+b} = \underline{cn+d}$

$cn+d = 3(c(n-1)+d) + 2n$

$cn+d = (3c+2)n + 3d - 3c = (3c+2)n + (3d-3c)$

$c = 3c+2 \rightarrow c = -1$

$d = 3d - 3c \rightarrow d = -\frac{3}{2}$

$\therefore a_n^{(p)} = -n - \frac{3}{2}$

$\therefore a_n = a_n^{(h)} + \underline{a_n^{(p)}} = \alpha 3^n - n - \frac{3}{2}$. fix α with $a_1 = 3$.

$3 = \alpha\cdot 3 - 1 - \frac{3}{2}$ $\therefore \alpha = \frac{11}{6}$

$\frac{11}{2} = 3 + \frac{5}{2} = 3\alpha$

$\therefore a_n = \frac{11}{8} \cdot 3^n - n - \frac{3}{2}$

$a_1 = \frac{11}{8} \cdot 3 - 1 - \frac{3}{2} = \frac{11-2-3}{2} = \frac{6}{2} = 3$

$a_2 = \frac{11}{8} 3^2 - 2 - \frac{3}{2} = \frac{33-4-3}{2} = \frac{26}{2} = 3 \cdot 3 + 2 \cdot 2 = 13$ verify

$a_3 = \cdots$

basis, recur. $\Rightarrow$ proof

Return to Tower of Hanoi.

$H_n = 2H_{n-1} + 1 \qquad H_1 = 1$

Sol) i) $k=1$. $r=2$ with mult. 1. $\therefore a_n^{(h)} = \alpha \cdot 2^n$

ii) $F(n) = 1$. $\quad H_n^{(p)} = C \quad C = 2C + 1 \quad \therefore C = -1$

$a_n = a_n^{(h)} + a_n^{(p)} = \alpha 2^n - 1$ with $H_1 = 1$

$1 = \alpha \cdot 2 - 1 \qquad \therefore \alpha = 1$

$\therefore H_n = 2^n - 1$

※ $F(n)$ … polynomial eq. Thm6

$F(n)$ … exponential eq