# Chap 7. generalized Recurrence Relation



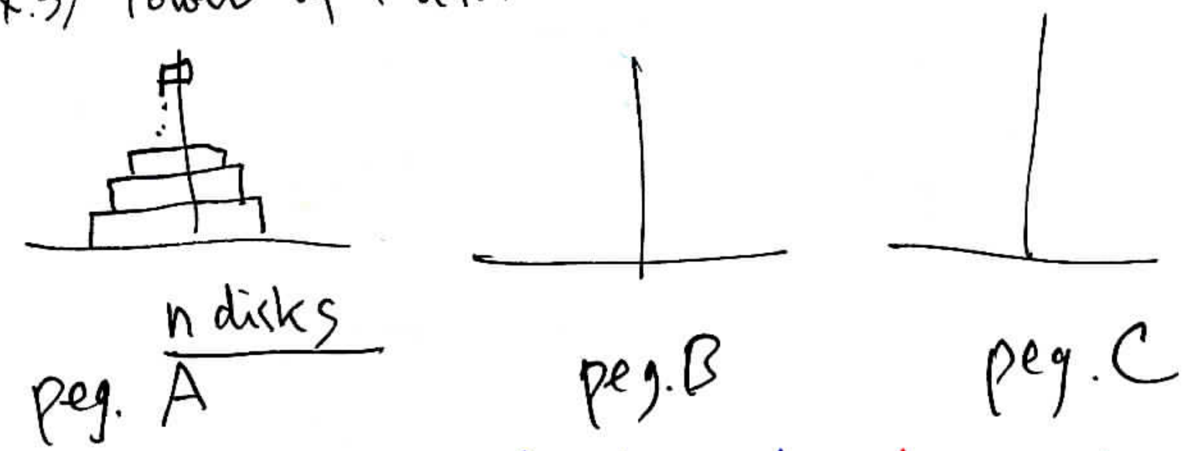
$a_n = f(a_{n-1}, a_{n-2}, \dots, a_{n-k})$ → recurrence relation with degree k

Ex $a_n = a_{n-1} - a_{n-2}$ where 2 basis

Ex2. $P_n = P_n + 0.06 P_n = 1.06 P_n$

Ex. $f_n = \underbrace{f_{n-1}}_{\text{young rabit}} + \underbrace{f_{n-2}}_{\text{old rabits}}$ $f_0 = 0,\ f_1 = 1$. degree 2

Ex.5) Tower of Hanoi




$H_n = 2H_{n-1} + 1$ when $H_1 = 1$ degree 1. $H_{10} = ?$
$= 2H_9 + 1$
$= 2(2H_8 + 1) + 1$

$H_n$ .. closed form

$H_n = f(H_{n-1})$ — rec. form

$H_n = f(n)$ _ closed form

Two methods

$H_1 = 1$, $H_2 = 2 \cdot H_1 + 1 = 2 \cdot 1 = 3$, $H_3 = 7$ $H_4 = 15$, $H_5 = 31$

$\boxed{\text{guess } (H_n = 2^n - 1)}$ $n \ge 1$ ($\cancel{n \ge 0}$)

i) basis $n = 1$ $H_1 = 2^1 - 1 = 2 - 1 = 1$

ii) assum $H_n = 2^n - 1$ (I.H.)

$$H_{n+1} = 2 \cdot \underbrace{H_n + 1 = 2 \cdot (2^n - 1)}_{\text{I.H.}} + 1 = 2^{n+1} - 2 + 1 = 2^{n+1} - 1$$

3) plug & chug !

$H_n = 2H_{n-1} + 1$

$= 2(2H_{n-2}+1)+1 = 2^2 H_{n-2} + 2 + 1$

$= 2^2(2H_{n-3}+1) + 2 + 1 = 2^3 H_{n-3} + 2^2 + 2 + 1$

$= 2^3(2H_{n-4}+1) + 2^2 + 2 + 1 = 2^4 H_{n-4} + 2^3 + 2^2 + 2 + 1$

$\vdots$

guessing

$= 2^{n-k-1} H_{k+1} + 2^{n-k-2} + \cdots + 2^0.$

$= 2^{n-k-1}(2H_k+1) + 2^{n-k-2} + \cdots + 2^0$

$= 2^{n-k} H_k + \underbrace{2^{n-k-1} + 2^{n-k-2} + \cdots + 2^0}$

$\vdots$

$= 2^{n-1} H_1 + 2^{n-2} + 2^{n-3} + \cdots + 2^0$

$= 2^{n-1}(1) + \cdots$

$= 2^{n-1} + \cdots + 2^0 = \frac{2^n - 1}{2-1} = \underline{\underline{2^n - 1}}$

구구단 : $5 \times 7 = 5+5+5+5+5+5+5 = 35$

Table matching (Pattern Matching)

9.2. Linear homogeneous R.R with degree k (Constant Coeff)

$a_n = \sum_{i=1}^{k} c_i a_{n-i} \quad (n \geq k)$

$\underbrace{c_1 \cdots c_k}_{k\text{-unknown}} \in \mathbb{R}$ $\quad$ $\underbrace{a_0 \cdots a_{k-1}}_{k\text{-known} \Leftrightarrow k\text{-basis}} \in \mathbb{R}$

$\underline{a_0, a_1 \cdots, a_{k-1}}$ - are know

$a_k = c_1 \cdot a_{k-1} + c_2 a_{k-2} + \cdots + c_k a_0$

3) Fibonacci

$$r = \frac{1 \pm \sqrt{5}}{2} \quad \text{with } f_0 = 0 \text{ and } f_1 = 1$$

$$f_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$f_0 =$ 0

$f_1 =$ 1

$f_2 =$ 1

$f_3 =$ 2

$f_4 =$ 3

$f_5$ 5

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