4/14 (与) Recursion
Chap. 4.

Recursion ... infinite wisdom
↳ Only way to express infinite set
in a finite way!

Math. Ind. $P(x)$ $x \in \mathbb{N}$. ($P(0)$ is true
$P(1)$ " false
⋮)

$P : \mathbb{N} \xrightarrow{=\{0,1,2,\dots\}} \mathbb{B}$
$\mathbb{B} = \{t, f\}$

We want to prove that $P(x)$ is true for $\forall x \in \mathbb{N}$.

Ex) $(x+1)^2 = x^2 + 2x + 1$

i) Basis : $P(0)$.
ii) Recursion (induction) : $P(n) \Rightarrow P(n+1)$. $\forall n \geq 0$.
($P(n) \Rightarrow P(n+1)$ $\forall n \geq 0$)
⇓
$\therefore P(n)$ $\forall n \geq 0$ $(n \in \mathbb{N}_0)$

i) $P(n_0)$
ii) $P(n) \Rightarrow P(n+1)$ $n \geq n_0$
↓
$P(n)$. $\forall n \geq n_0$

$P(0) \wedge P(1) \wedge \dots \wedge P(n)$

Proof
i) $P(0)$. by rule i) (Basis)
ii) $P(0) \Rightarrow P(1)$. by the rule ii) recu. for $n = 1$
iii) $P(1) \Rightarrow P(2)$. " " $n = 2$
⋮

Set
. $\{1, 2, \dots, n\}$ ... finite set
① $\{1, 2, \dots\}$ ... infinite set. (iteration)
② $1 \in \mathbb{N}$ (recursion)
$n \in \mathbb{N} \Rightarrow (n+1) \in \mathbb{N}$

To prove $\Downarrow p \Rightarrow q \Uparrow$

$p' \Rightarrow p$

$p \Rightarrow p''$

$p' \Rightarrow q$ easiest

$p \Rightarrow q$

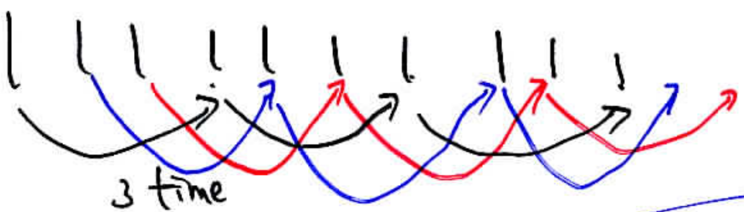
$p'' \Rightarrow q$ hardest

Generalized induction

$$\left.\begin{array}{l} \text{i) (1) } P(c). \\ P(c+1). \\ \vdots \\ \text{(n) } P(c+C-1). \end{array}\right\} \rightarrow C\text{-bases}.$$

ii) $P(n) \Rightarrow P(n+C) \quad \forall n: n \geq c$

$P(n). \ \forall n \geq c.$



$\Sigma = \{0,1\}$

$\Sigma^0 \triangleq_b \{\varepsilon\}$

$\Sigma^1 \triangleq_r \Sigma^0 \cdot \Sigma = \{\varepsilon\}.\Sigma \ (=\Sigma)$

$= \{\varepsilon.0, \varepsilon.1\} = \{0.1\}$

$\Sigma^2 \triangleq_r (\Sigma^1)\Sigma \triangleq_b (\Sigma^0 \cdot \Sigma)\Sigma$

$=_b \{\varepsilon\} \cdot \Sigma \cdot \Sigma = \Sigma \cdot \Sigma = \Sigma^2$

$= \{00, 01, 10, 11\}$

$\Sigma^3 = \ldots$

$\Sigma^4 = \{000, \ldots, 0111\}$

$= \{0000, \ldots, 1111\}$

i) $\varepsilon \in \Sigma^*$

ii) $w \in \Sigma^* \wedge a \in \Sigma \Rightarrow wa \in \Sigma^*$

Ex. 2 $\varepsilon = \lambda$

i) $\varepsilon \in \Sigma^*$ $\quad \Sigma = \{a.b\}$ (i) 1

ii.1) $\varepsilon \in \Sigma^*, a \in \Sigma \Rightarrow a \in \Sigma^*$ (ii) 2

$\varepsilon \in \Sigma^*, b \in \Sigma \Rightarrow b \in \Sigma^*$ (")

$$\left.\begin{array}{l} \text{ii.2) } a \in \Sigma^*, a \in \Sigma \Rightarrow aa \in \Sigma^* \ (") \\ \quad " \quad b \in \Sigma \Rightarrow ab \in \Sigma^* \ (") \\ b \in \Sigma^*, a \in \Sigma \Rightarrow ba \in \Sigma^* \\ \quad " \quad b \in \Sigma \Rightarrow bb \in \Sigma^* \end{array}\right] 4$$

$\left(\varepsilon \in \Sigma^*, a \in \Sigma \Rightarrow a \in \Sigma^*\right)$

"

No addition to set Σ^*

ii.3) . $\quad aaa \in \Sigma^*$ $\vdots$ $bbb \in \Sigma^*$ 8 ... 2^n

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \cdots$

$= \bigcup_{i \in \mathbb{N}_0} \Sigma^i$

where $\Sigma^i : \begin{cases} \Sigma^0 =_b \{\varepsilon\} \\ \Sigma^i =_r \Sigma^{i-1} \cdot \Sigma. \end{cases}$