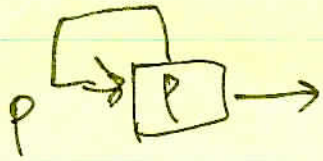
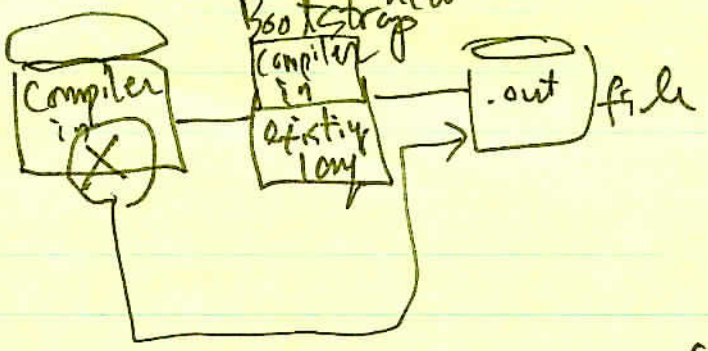
4/12(A) Big-O notation

Halting problem
A program whose data is the program itself.

∴ Compiler for long. (X)





3.2 Big-O notation

Def: Let $f: \mathbb{N} \to \mathbb{R}$ or $\mathbb{R} \to \mathbb{R}$

We say $f(x) \in O(g(x))$ if

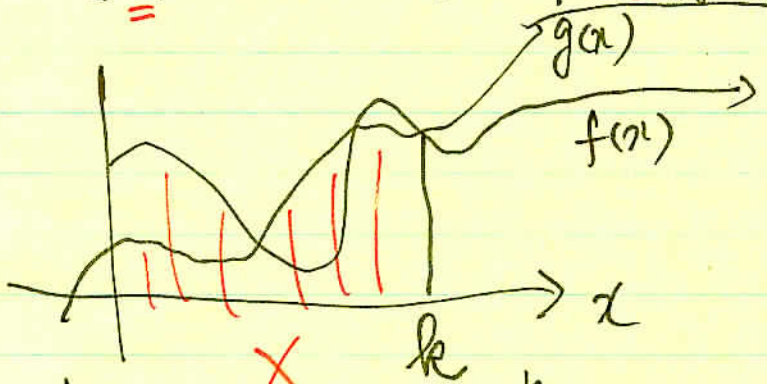
$$\exists C, k \in \mathbb{R} \ \ni . \ |f(x)| \leq C \cdot |g(x)| \quad \forall x > k$$

$O: F \to 2^F$

$\to O: 2^F \to 2^F$

$O(f)$: —

$O(F) = \bigcup_{f \in F} O(f)$



24%
19% ~~3~~ 5
80% ~~22~~ 24
24
30(6)

$g(x)$: $1, x, x^2, \dots, x^k, \dots, x^x, \dots$

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ -- polynomial.

~~$O(f(x))$~~ $\to f(x) \in O(g(x))$ → $g(x)$: $x^{n+1}, x^{n+2}, \dots$; $x^{n-1}, x^{n-2} \dots$ X ; $x^n \dots$ OK

c.k. ... witness for fn

x^n

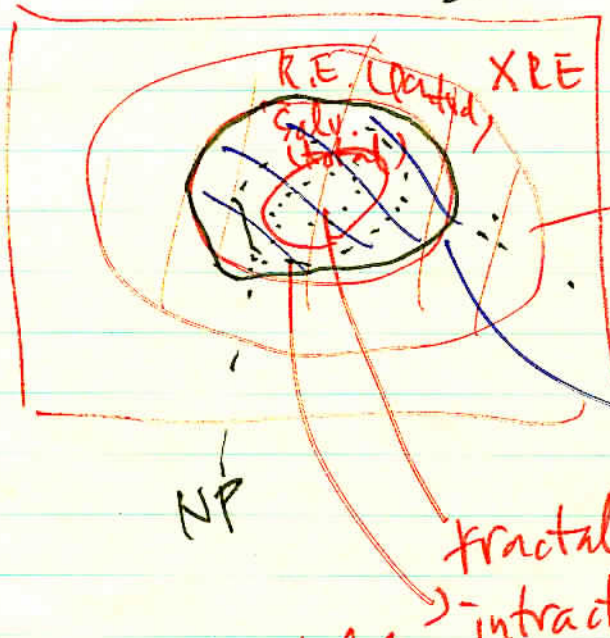
$n!$ Not $n! \in O(n!)$ but $n! \in O(n^n)$ $n! \notin O(n^{n-1})$

Eventhough $n! \leq n^n$

$1, \log n, n, n\log n, n^2, n^3, \ldots, n^k$ ... polynomial (tractable)

$2^n, n!, n^n$ vs $|\mathbb{N}|, |\mathbb{N}|^2 \ldots |\mathbb{N}|^k$ countable

$2^{|\mathbb{N}|}$ .. uncountable .. exponential (intractable)



Programmable (undecidable)

Algorithm (decidable)

tractable
intractable

P: problem → tractable
↘ intractable — ... is not decidable

Halting prob ... exist / not

※ Problem P:
∃ algorithm A ∋. $O(f(n))$ .. $f(n)$ is a upper bound for the problem P.

NP: Nondeterministic Polynomial.

U.B. $O(k^{P(n)})$

NP-complete ... the hardest one among NP.