1) 6/8 Formal Languages & Automata Theory CS 322 1

Grammar $G = (N, T, P, S)$ (V, T, P, S)

N : a finite set of Nonterminal symbols

T : a " " ~~Voca~~ Terminal " ; where $N \cap T = \emptyset$.

we ~~use~~ $V = N \cup T$.

$P \subseteq V^+ \times V^*$: a finite set of pairs called as productions. $\alpha \in V^+, \beta \in V^*$ if $(\alpha, \beta) \in P$, we write $\alpha \to \beta$.

$S \in N$: a distinguished symbol, called start symbol.
(nonterminal) (axiom)

A grammar is called context-free, if $P \subseteq N \times V^*$.

if $(A, \alpha) \in P$, we write $A \to \alpha$.

We define $\Rightarrow \subseteq V^* \times V^*$ using $P = \to \subseteq V^* \times V^*$

derivation

$\gamma \alpha \delta \Rightarrow \gamma \beta \delta$, $\gamma, \delta \in V^*$, if $\alpha \to \beta \in P$ $(\to)$

Ex) $S \to 0S1 \mid \varepsilon$. $(S \to 0S1, S \to \varepsilon)$

$\{(S, 0S1), (S, \varepsilon)\} = P$.

$S \Rightarrow \varepsilon$ $(\gamma, \delta = \varepsilon)$

$\Rightarrow 0S1 \Rightarrow 01$ $(\gamma = 0, \delta = 1)$

$\Rightarrow 00S11$ (")

$\{0^n 1^n \mid n \geq 0\}$

$R \subseteq A \times B$
if $(a, b) \in R$
aRb
we write.

$R^* = \bigcup_{i=0}^{\infty} R^i$

$\to$, $\Rightarrow$, $\Rightarrow^*$.

(induced relation)

~~$L(G) = \{x \to\}$~~

$G = (N, T, P, S)$ $L : 2^G \to 2^{T^*}$.

V: $\boxed{N_{.S} \mid T}$

$L(G) = \{x \in T^* \mid S \Rightarrow^* x\}$ N. Chomsky.

2)

$B \to B \wedge B \mid B \vee B \mid \neg B \mid (B)$

$\mid v \mid \mathbb{T} \mid \mathbb{F}$

$G = (N, T, P, S)$

$N = \{B\}$

$T = \{\wedge, \vee, \neg, (,)\ v, \mathbb{T}, \mathbb{F}\}$

$P = \{ \dots \}$

$S = B$

P8.5. Def. 3

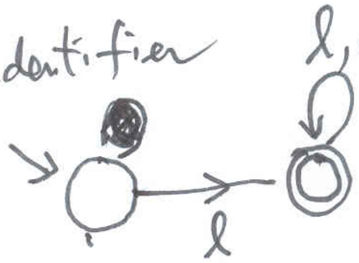
We define finite state machine (automaton) $M = (S, I, f, s_0, F)$

1. S: a finite set of states
2. I: a " " of input symbols (terminal)
3. $f: S \times I \to S$ .. a finite set of transition function
4. $s_0 \in S$: a distinguish state called as start state.
5. $F \subseteq S$: a set of final states. $\vdash \$ \dots$

Ex) Identifier

l: letter $= \{a, \dots z\}$

d: digit $= \{0, \dots 9\}$



$f: S \times I \to S$

$\hat{f}: S \times I^* \to S$.

$s \in S$

$\hat{f}(s, \varepsilon) = s$

$\hat{f}(s, xa) = f(\hat{f}(s, x), a)$

$x \in I^*, a \in I$

$(xa \in I^+)$

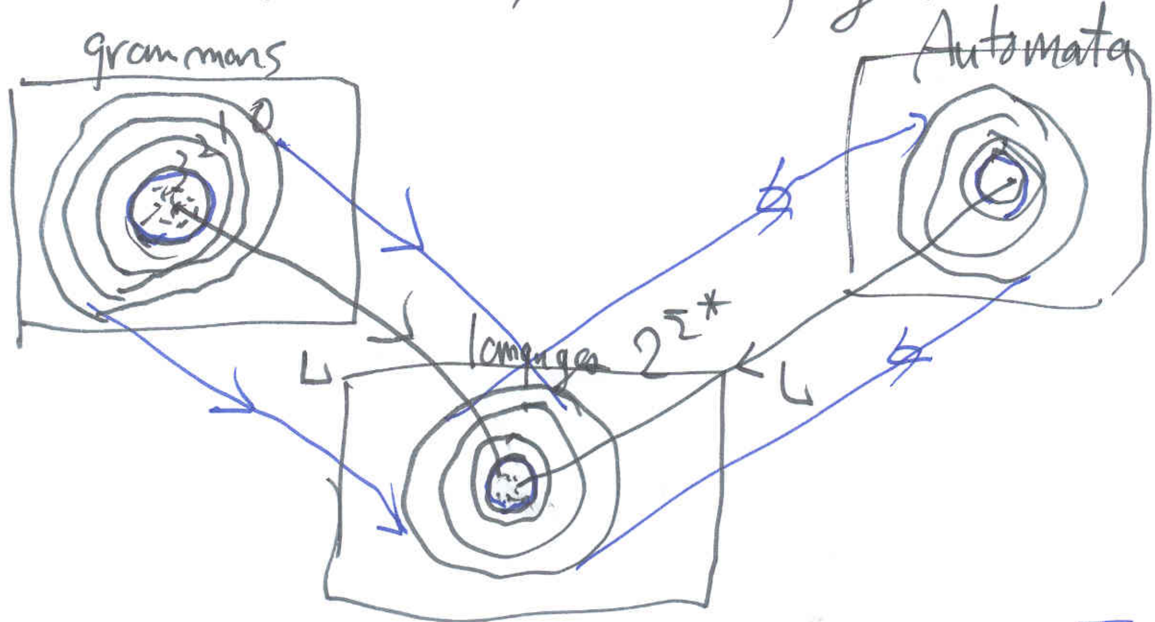
$L(M) = \{x \in I^* \mid \hat{f}(s_0, x) \in F\}$

S

Consider $\Rightarrow$ vs f

$\Rightarrow^*$ vs $\hat{f}$

3) grammar, automata, and languages



Chomsky's Hierarchy (Grammar / language / automata) $\longleftrightarrow$ Turing Machine

f.a + memory (tape)

1. Type 0 : Recursively enumarable ~~(No r~~
   (no restriction of P)

2. Type 1 : Recursive … terminate
   ~~(context-sensitive) $\alpha_1 A \alpha_2 \to \alpha_1 \beta \alpha_2 \in P$ $A \in N$ $\alpha_1, \alpha_2, \beta \in V^*$~~

   TM that always terminates.

3. Type 2. Context-free grammar
   $A \to \beta \in P, \quad A \in N, \beta \in V^*$ $\longleftrightarrow$ f.a. + stack
   pushdown automata

3. Type 3 Regular grammar
   $A \to aB$, or $a \in P$ $\longleftrightarrow$ finite state automata
   $A, B \in N, \ a \in T$.

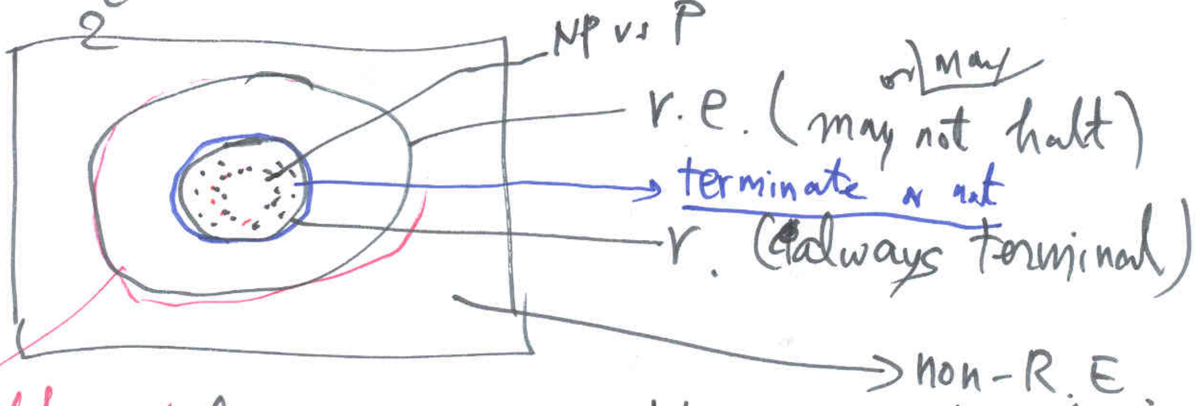
   ↑ no memory except ~~stack~~ state.

Turing Machine $\longleftrightarrow$ Computable

Turing - Church's ~~Theo~~ Thesis

↳ Recursive function

2^{Σ^*}



countable vs uncountable

language = problem

↳ halting problem.

$L \subseteq \Sigma^*$ $\qquad f: D \longrightarrow \{0,1\}$

$L \in 2^{\Sigma^*}$ $\qquad \{0,1\}^{|D|}$

$|2^{\Sigma^*}| = 2^{|\mathbb{N}|}$ $\qquad = 2^{|\mathbb{N}|}$

uncountable

Program $\in \Sigma^*$

countable