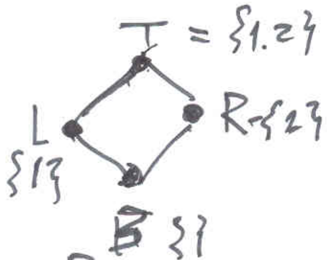
$(A, \vee, \wedge)$ algebraic system defined by a lattice $(A, \leq)$ (poset)

Example $(\mathbb{N}, \max, \min)$ 〃 $(\mathbb{N}, \leq)$

$(\{t, f\}, t \geq f)$



		$\wedge$	$\vee$
t	t	t	t
t	f	f	t
f	t	f	t
f	f	f	f

$\vee$	T	L	R	B
T	T	T	T	T
L	T	L	T	L
R	T	T	R	R
B	T	L	R	B

$\Rightarrow$ $\Leftarrow$ $\equiv$

$\vee$	

$\neg$	
T	B
L	R
R	L
B	T

Distributive lattice

$A \Rightarrow B \equiv (\neg A) \vee B$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$0 \in A$ is called universal lower bound, if (top)

$\forall a \in A \quad 0 \leq a$

$1 \in A$ 〃 upper (bottom)

$\forall a \in A \quad a \leq 1$

$(2^A, \subseteq)$ $A \ldots$ top $\{\}$ bottom

$B \subseteq A$ or $B \in 2^A$

$0 \vee a = a$, $\quad B \cup \emptyset = B$

$0 \wedge a = 0 \quad B \cap \emptyset = \emptyset$

$1 \vee a = 1$ $\quad$ ⋮

$1 \wedge a = a$

$(A, \leq)$ $0, 1$

$a \vee b = 1$, $a \wedge b = 0$

b is called ~~then~~ complement of a.

$b = \neg a$

$\neg : A \to A$

$\vee : A \times A \to A$

$\wedge : A \times A \to A$

$\underline{(A, \vee, \wedge, \neg, 0, 1)}$ dist. complent $>$ Boolean ~~algebra~~ lattice

~~(A)~~

$(2^A, \cup, \cap, \bar{\ }, A, \emptyset)$ is a <u>boolean algebra</u>

finite boolean alg. : 2^n elements

[illegible]

≡	B	L	R	T
B	T	R	L	B
L	R	T	B	L
R	L	B	T	R
T	B	L	R	T

– [illegible]

Chap 12 Modeling Computation

12.1 Language and grammars. N. Chomsky

Def 1. <u>Vocabulary V</u> : a finite, nonempty set of elements called symbol.

<u>$a \in V$: symbol</u>

$x \in V^*$ = string (a sequence of symbols)

$L \subseteq V^*$: a set of strings (language)

Ex) Compiler : token (syntax analysis (parsing))
ASCII code (lexical analysis)

① syntax analysis

② semantics ″

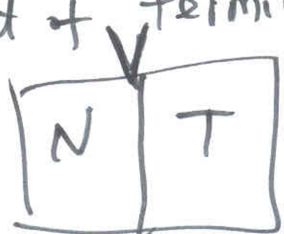
③ code generation (optimization)

Def2. A phrase-structured grammar.

$G=(V,T,S,P)$

V : vocabulary symbols $\qquad N\cap T=\emptyset$

T : a set of terminal symbols $\qquad V-T=N$ nonterminal symbol



$V=N\cup T$

$S\in N$: a start symbol

$P\subseteq V^*\times V^*$: a set of production rules

$(\alpha,\beta)\in P$ $\quad \beta\in V^*$, $\alpha\in V^+$ $\quad$ written $\alpha\to\beta$

Context free gram

$P\subseteq N\times V^*$ $\qquad A\in N,\ \alpha\in V^*$

$A\to\alpha\in P$ (~~A~~

$(A,\alpha)\in P$

derivation

$\gamma\alpha\delta\Rightarrow\gamma\beta\delta$ if $\alpha\to\beta\in P$, $\gamma,\delta\in V^*$.

$\Rightarrow\subseteq V^*\times V^*$

$\Rightarrow^*\subseteq$ ″ $\qquad$ Let $G=(V,T,S,P)$ be a grammar.

$L(G)=\{x\in T^*\mid S\Rightarrow^* x\}$: language generated by G.

Ex) $S \to 0S1 \mid \varepsilon$

$V = \{0, 1, S\}$, $T = \{0, 1\}$,

$L(G) = \{0^n 1^n \mid n \geq 0\}$

$S \Rightarrow \varepsilon$

$S \Rightarrow 0S1 \Rightarrow 01$

$\Rightarrow 00S11 = 0011$

$\Rightarrow 000S111$

$P \to 0P0 \mid 1P1 \mid \varepsilon \mid 0 \mid 1$

$E \to E+E \mid E*E \mid a \mid (E)$

$B \to B \wedge B \mid B \vee B \mid \neg B \mid u \mid (B)$