5/23 Shortest Path, Trees

Edge

Weighted graph $f: E \to R^+$

weight of path $f^*: E^* \to R^+$

E^n

$V^* \quad V^{n+1}$

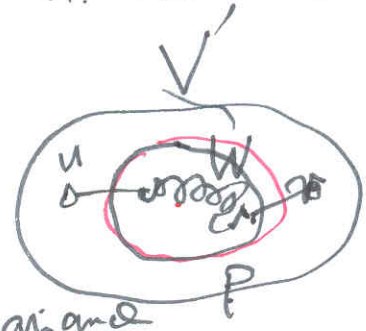
Def Let $W \subseteq V$. $u, v \in V$, $L_f^W(u,v) = f(u, w_1, w_2, \dots w_n, v)$ s.t.

$(u, w_1, w_2, \dots w_n, v)$

$f(u, w_1, w_2 \dots w_n, v) \le f(u, x_1, x_2 \dots x_m, v)$

$n, m \ge 0$, $1 \le \forall i \le n$, $(\quad " \quad) \in V^*$

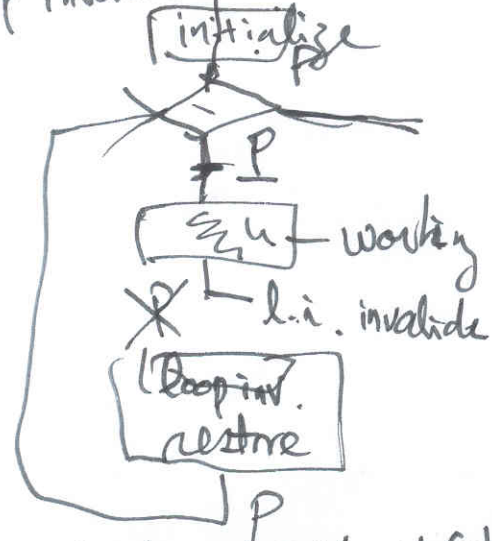
$w_i \in W$, $1 \le \forall i \le m$, $x_i \in W$



$1 + \dots + 100 \to 1 + \dots + i$

$i = 100$

Loop invariant



$i := 0; S := 0$

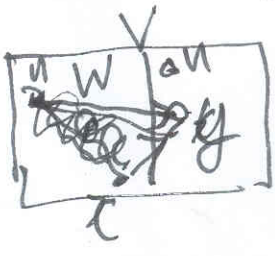
do $i \le n \to$ $S = 1 + \dots i$

$S := S + i;$

$i := i + 1$

od

Thm. Let $W \subseteq V$, $u \in V$, $x \notin W$, $L_f(u,x)$ is the minimum among $\forall y \in W$,



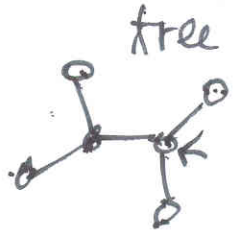
we get

$L_f(u,x) = \min\{L_f^W(u,y), L_f^W(u,x) + f(x,y)\}$

A Discipline of Programming E.W. Dijkstra

Chap. 10 Trees

Def. 1. ugraph $\exists$ unique path $\Leftrightarrow$ for every pair of vertices.
connected acyclic graph



rooted tree

Def 2 A rooted tree ... root, (V, E, r)

$r \in V$: root.

Acyclic graph $=$ forest