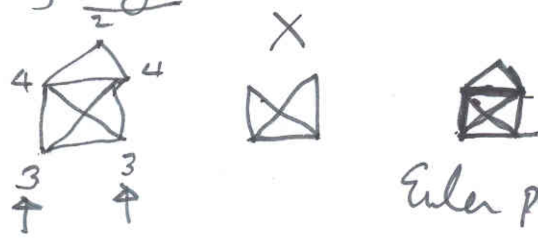
1) 5/21. Euler, Hamilton Circuit/Path
cycle
Shortest Path 1

P633

Euler circuit : circuit (cycle) every edge in the graph : 한붓그리기
path path



Euler circuit $\Leftrightarrow$ every vertex has even degree

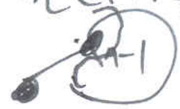
Euler path $\Leftrightarrow$ two vertices of odd degree.

Hamilton circuit : circuit every vertex
path path

Hamilton circuit : Traveling Sales man

$1 \cdot (n-1)(n-2) \ldots 1$ $(n-1)!$
(n)
$n!$

n cities



$n \cdot (n-1)(n-2) \ldots 1 = n!$

~~Traveling sales~~

Shortest path

$G = (V, E)$ $f : E \to R^+$: Edge weighted graph
labelled

cost : edge
path

path $(v_0, v_1, v_2, \ldots, v_n)$ $v_i \in V$.

$n \geq 0$ if $n=0$ (v_0) $f^*(v_0, v_0) = 0$

$n > 0$ $f^*(v_0, v_1, \ldots v_n) = \sum_{j=0}^{n-1} f(v_j, v_{j+1})$

$= f(v_0, v_1) + f(v_1, v_2) + \cdots + f(v_{n-1}, v_n)$

$f^*(v,v)=0$ ... basis

$f^*(v,u)=f^*(v,x)+f(x,u)$ where $\{x,u\} \in E$.

2 paths - 2 costs

$f: E \to R^+$ $\qquad f \subseteq f^*$

$f^*: E^* \to R^+$ $\qquad$ extension

Def $\min_f(\underline{u}, v) = (v_0, v_1, \dots, v_n)$

$$f(\underline{v_0, \dots v_n}) \le f(\underline{u_0, \dots, u_m}) \quad \forall\, \underline{(u_0, \dots, u_m)} \text{ s.t. } u_0 = u,\ u_m = v$$

$v_0 = u$

$v_n = v$ $\qquad$ minimum.