5/18 Planar graph.

Subgraph $G'=(V',E')$ of Graph $G=(V,E)$

$\rightarrow \left[\begin{array}{l} V' \subseteq V \\ E' \subseteq E \end{array} \right.$ vs. $E' \subseteq E$, $V = \{u, v \mid \{u, v\} \in E'\}$

conn. or discon.   connected

n-cube, revisited.

$Q_0 = \{ \cdot \ \cdot \ )$

$Q_n = (V, E)$ $V = \{v_0, \dots v_{2^n-1}\}$ # edge = ?

$\updownarrow f$

$Q_n' = (V', E')$ $f : V \leftrightarrow V'$ $V \cap V' = \emptyset$

$f : E \leftrightarrow E'$

$V' = \{v' \mid f(v) = v', v \in V\} \Rightarrow V' = f(V)$

$E' = \{\{f(v), f(u)\} \mid \{v, u\} \in E\}$

$f(\{v, u\}) = \{f(v), f(u)\}$ $E' = f(E')$

$Q_{n+1} = (V \cup f(V), E \cup f(E) \cup \{v, f(v)\})$

k-connected (two vertices, graph)

remove (k-1) edges still connect.

k " disconnected

simple cycle — 2 connected

K_n : k-1 connected

Tree — connected acyclic graph

$\exists$ unique path between pair of two vert.

$|E| = |V| - 1$ (if con $|E| \geq |V| - 1$)

→ minimal edges for connection

3) Thm Every connected graph has spanning tree

T : connected spanning subgraph of G with smallest # of edges.

assum cycle C $(v_0, v_1, \ldots \underbrace{v_n, v_0})$

x, y .. path $\{v_n, v_0\}$

$\times \{v_n, v_0\}$ $\underline{v_0, \ldots v_n}$

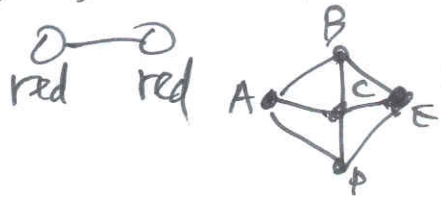
smallest, connected $\longrightarrow$ contradiction

$\therefore$ acyclic

$\therefore$ tree.

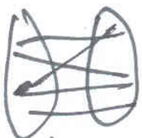
9.8 Graph coloring.

Def 1. A graph G is a k-colorable, X map coloring



$|C|^{|V|}$

Bipartite graph



2-colorable

Complete graph K_n

n-colorable n-vertex

for $A \in C$ do

for $B \in C$ do.

⋮

for $(A, B, C, D, E) \in C^{5}$ do

if diff(AB) ∧

diff(A C) ∧

⋮

|E|

diff(A,B) ∧ diff(A,C). ∧

...

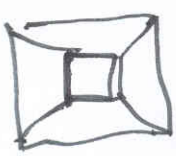
9.7 Planar Graph

Def. A

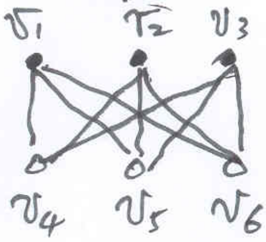
K_4 $\xleftrightarrow[\text{onto}]{1:1}$

planar representation

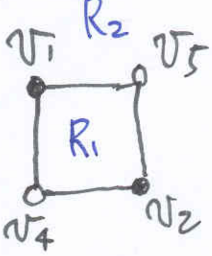
Q_3 $\xleftrightarrow[\text{onto}]{1:1}$



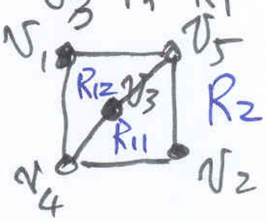
region (face)



$K_{3,3}$



v_3 in R_1



v_6 in R_{11}

R_{12}

R_2

$K_{3,3}$ is not planar.

3) G connected planar graph

v vertices
e edges
r regions

$$r = e - v + 2$$

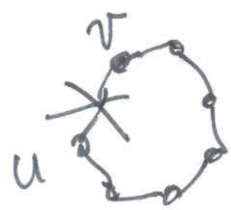
proof. Induction on # of edges.

basis $e=0,\ v=1,\ r=1$. $e-v+2 = 1 = r$

induction Consider conn. planar graph G with $e+1$ edges

1. If G is acyclic. G is a tree. $e = v-1,\ r=1$
   $\therefore\ r = e - v + 2$ is OK

2. If G has a cycle C.
   consider $\underbrace{\{u, v\}}_{\text{an edge}}$ and a spanning tree T $\{u,v\}$ is not in T.
   in C



Remove edge $\{u,v\}$ from G, it is called as G'.

G' is connected planar graph with e edges

$r = e - v + 2$ (by I.H.)

G $r+1$ region, $e+1$, v,

$(r+1) = (e+1) - v + 2$ $\therefore$ O.K.

Chromatic number of G ... minimum # of colors

$\chi(G)$ $\chi(K_n) = n$ $\chi(\text{tree}) = 2$

$\chi(\text{Bipartite}) = 2$

$\chi(\text{path}) = 2$

$\chi(\text{cycle}) \Leftarrow$ even 2, odd 3.

Bipartite graph.