1) 5/16 Tree (ugraph) (X rooted tree digraph)

Tree $\overset{\text{Def}}{\Leftrightarrow}$ acyclic connected graph

$\Leftrightarrow \exists$ unique path between every pair of vertices

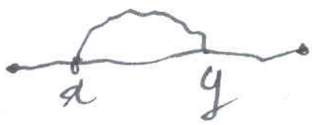
$\Leftrightarrow |V| = |E| + 1$

Defition of tree

Thm 1 properties of tree vs characteristics of tree

1 Subgraph of a tree is also a tree.

2 unique path



cycle.

3 $\nexists$ T + edge → cycle

u v

path

path

$(v_0, \dots v_n)$ $n \geq 0$

		edge
n=0	(v_0)	
n=1	(v_0, v_1)	$\{v_0 v_1\}$
n=2		
3		

4. Tree − edge → disconnected

delete $\{u, v\} \in E$ u. v .. unique path = edge

$x^n, n \geq 1$

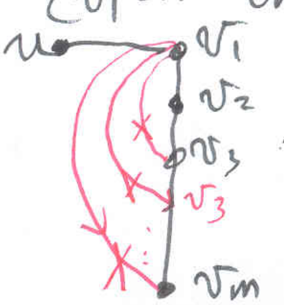
x

$x \cdot x = x^2$

$x \cdot x \cdot x = x^3$

5. $|V| \geq 2$ → at least two leafs.

$(v_1 \cdots v_m)$ is the longest path in the tree $(m \geq 2)$



: ~~v_1 is a~~

$\{u, v_1\} \in E$, u is not in the path.

v_1 is a leaf.

$x^0 = 1$

String over V

$x \in V^*$

$a \in V$ ↑ symbol

6. $|V| = |E| + 1$ ... minimal connection

$|E| \leq |V| - 1$

↓ symbol a

ε

a_1

$a_1 a_2$

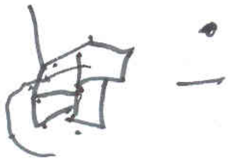
$a_1 a_2 a_3$

String

$\boxed{\text{Spanning subgraph } V'=V}$

subgraph $G=(V,E)$ $G'=(V',E')$ $V'\subseteq V, E'\subseteq E$

$E'\subseteq E,\ V'=\{u,v \mid \{u,v\}\in E'\}$

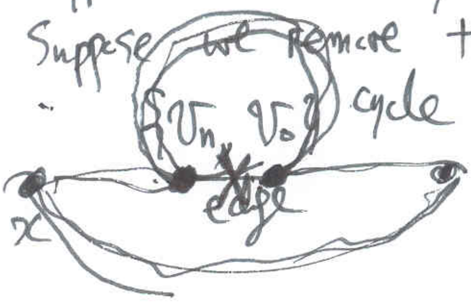


Every connected graph has spanning ~~graph~~ tree.

proof Let T be a spanning ~~tree~~ subgraph with the smallest # of edges

Suppose T has a cycle $(v_0, v_1, \dots v_n, v_0)$

Suppose we remove the edge $\{v_n, v_0\}$



$x, y \in T$.

~~edge~~ Tree.

T is acyclic

T is a spanning tree.