

5/17 Generating function  
closed form

$$1, a, a^2, \dots \quad \underbrace{1 + ax + a^2x^2 + \dots}_{\uparrow |-(ax)|} = \frac{1}{1-(ax)}$$

$$(1, 1, 1, \dots) \quad \Leftrightarrow \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

↓ diff.

$$(\underline{1, 2, 3, \dots, n+1}) \quad \Leftrightarrow \frac{d}{dx} \frac{1}{1-x} = \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \dots$$

$$(\underline{0, 1, 2, \dots, n}) \quad \Leftrightarrow x \cdot \frac{1}{1-x^2} \quad \downarrow \text{shift right}$$

$$= \underline{0 + x + 2x^2 + 3x^3 + \dots}$$

$$(f_0, f_1, \dots, f_k, f_{k+1}, \dots) \quad \leftarrow \text{shift left } k \text{ times}$$

$$(f_k, f_{k+1}, \dots) \quad \leftarrow \text{Add zero (shift right) } j \text{ times}$$

$$(0^j, 0^j, \dots, 0^j, f_k, f_{k+1}, \dots)$$

Fibonacci sequence  $n \text{ places}$

$$(0, 1, 0, \dots)$$

$$(0, 0, f_1, f_2, \dots)$$

$$+) (0, 0, f_0, f_1, \dots)$$

$$(0, \underline{x}; f_1+f_0, f_2+f_1, \dots, f_{n-1}+f_{n-2})$$

degree 1

$$F(x) = \frac{x}{(1-x-x^2)} = \frac{x}{(1-\frac{1+\sqrt{5}}{2}x)(1-\frac{1-\sqrt{5}}{2}x)} \rightarrow \text{degree 2}$$

$$= \frac{A}{(1-\alpha_1 x)} + \frac{B}{(1-\alpha_2 x)} \quad A = \frac{1}{\sqrt{5}}, B = -\frac{1}{\sqrt{5}}$$

$$= \frac{\frac{1}{\sqrt{5}}}{1 - \frac{1+\sqrt{5}}{2}x} - \frac{1}{\sqrt{5}} \frac{1}{1 - \frac{1-\sqrt{5}}{2}x}$$

$$\therefore f_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2}\right)^n \quad C_n =$$

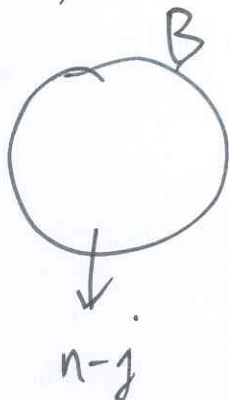
$$(a_0, a_1, a_2, \dots, a_{k-1}, 0, \dots)$$

$$\left( \begin{array}{ccc} 0 & \dots & 0 \\ \downarrow & & \downarrow \\ a_0 & a_2 & a_{k-1} \end{array} \right)$$

$$A(x) = \frac{\text{degrees } k-1}{\text{pol. degree } k}$$

$$\{a_j\} \leftrightarrow (1, 1, 0, \dots)$$

$$= ({}_1C_0, {}_1C_1, {}_1C_2, {}_1C_3, \dots)$$



$$A \cap B = \emptyset$$

$$a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \dots + a_n b_0$$

$$\left( \begin{array}{cccc} {}_1H_0 & {}_1H_1 & {}_1H_2 & {}_1H_3 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & 1 & \dots \end{array} \right) \left( {}_2H_0, {}_2H_1, {}_2H_2, {}_2H_3, \dots \right)$$