5/4 Linear recurrence relation

Thm 2. -double root r_0

$$a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$$

Ex. 5 $a_n = 6a_{n-1} - 9a_{n-2}$ $a_0 = 1$, $a_1 = 6$

$$r^2 - 6r + 9 = 0$$

$(r-3)^2 = 0$ — 중근

$$a_n = \alpha_1 3^n + \alpha_2 n 3^n$$

Thm 3 degree k, $\underline{r_1}, \underline{r_2} \cdots \underline{r_k}$ pairwise distinct root

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \cdots + \alpha_k r_k^n.$$

Ex. 6. C.E. degree 3 $1, 2, \frac{1}{3}$.

$$a_n = \alpha_1 + \alpha_2 2^n + \alpha_3 3^{-n}$$

Thm 4. degree k. $r_1, r_2, \ldots, r_t$ distinct root $(t \le k)$

multip. $m_1, m_2, \ldots, m_t$ $\sum_t m_t = k$

$$\alpha_n = (\alpha_{1,0} + \alpha_{1,1} n + \alpha_{1,2} n^2 + \cdots + \alpha_{1,m_1-1}) r_1^n$$

$= 1$

7/ Linear Nonhomogeneous R.R.

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + F(n)$$

$$a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$$ – associated homo. r.r.

$$a_n = \underbrace{a_n^{(h)}}_{\text{homogeneous solution}} + \underbrace{a_n^{(p)}}_{\text{particular solution}}$$

Ex. 10. $a_n = 3a_{n-1} + \underbrace{2n}_{=0}$ $\quad a_1 = 3.$

$a_n = 3a_{n-1}$

$r = 3$

$a_n^{(h)} = \alpha_1 3^n$ ... homo. solution

particular solution

$F(n) = 2n$ try $a_n^{(p)} = cn + d$

check recurrence

$\hookrightarrow cn + d = 3(c(n-1) + d) + 2n$

$\underbrace{(2c+2)}_{=0} n + \underbrace{(2d-3c)}_{=0} = 0$

$c = -1, \quad d = -\frac{3}{2}$

$\therefore a_n = \alpha 3^n - n - \frac{3}{2}$, $\quad a_1 = 3 \longrightarrow$ check basis.

$\alpha = \frac{11}{6}$

Ex. Tower of Hanoi

$H_n = 2H_{n-1} + 1 \qquad H_1 = 1$ (degree 1)

$r = 2 \qquad a_n^{(h)} = \alpha 2^n$

Try $a_n^{(p)} = C \qquad C = 2C + 1$ rec. rel.

$\therefore C = -1$

$a_n = \alpha 2^n - 1, \quad H_1 = 1$

$\alpha 2^1 - 1 = 1 \qquad \alpha = 1.$ $\quad \therefore \boxed{H_n = 2^n - 1}$

3) $a_n = a_{n-1} + 2a_{n-2} + 2^n$,

$r^2 - r - 2 = 0$

$(r+1)(r-2) = 0$

$r = -1,\ r = 2 \qquad a_n^{(h)} = \alpha_1 (-1)^n + \alpha_2 2^n$.

$(\_\_\_)\cancel{x^n}$

$\shortparallel$

1

$a_n^{(p)} = \quad C_1 (-1)^n + C_2\, n 2^n$

Generating ftn.

infinite seq. $\xleftrightarrow{1:1}$ ftn.