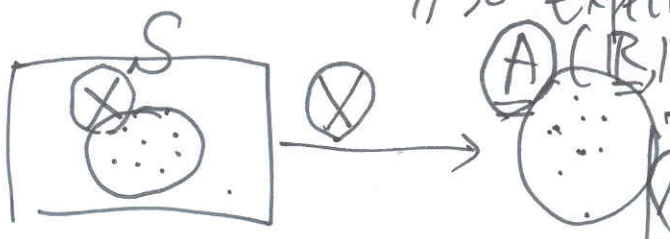
4/30 Expected value



$X: S \to A$ function

$(A, +)$ $+: A \times A \to A$: binary operation on A

$\forall a, b \in A$, $a+b \in A$ closed

→ algebraic system

$f: A \to B$: function

$f: A \to A$: unary operation

$f: A \times A \to A$: binary "

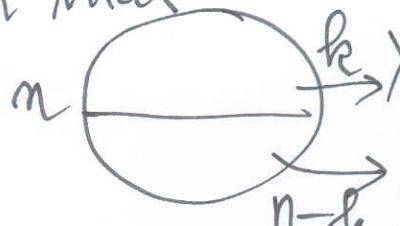
$$E(X) = \sum_{s \in S} p(s) \cdot X(s)$$

Thm 1 $X: S \to \mathbb{N}$

$$E(X) = \sum_{r \in X(S)} p(X=r) \cdot r$$

Thm 2 Bernoulli trial

n trial.



$p(X=k) = p$

$p(X \neq k) = 1-p = q$

$E_1: S \to A$, $E_2: S \to A$.

$E_1 + E_2: S \to A$

$$\sum_{j=1}^{\infty} j(1-p)^j = 1 \cdot (1-p) -$$

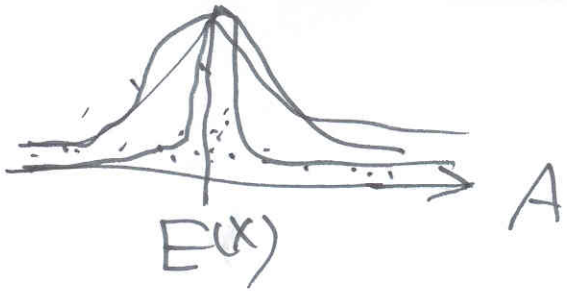
$$\sum_{j=1}^{\infty} j p^j = 1 \cdot p^1 + 2 \cdot p^2 + 3 \cdot p^3 + 4 p^4 + \cdots = \frac{1}{(1-p)^2} \quad |p| \leq 1$$

$$\sum_{j=1}^{\infty} p^j = p^1 + p^2 + p^3 + \cdots \quad |p| < 1$$

$$= \frac{1}{1-p}$$

$$p \cdot \sum_{j=1}^{\infty} j p^j = p^2 + 2p^3 + 3p^4 + 5p^k$$

$$(1-p) \sum_{j=1}^{\infty} j p^j = p^2 - \cdots$$

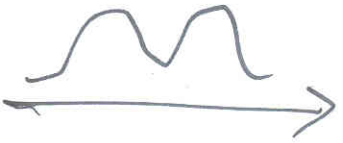


$X : S \to A$

(A+)

X

Thm 1
Thm 6



전수 points

Thm 6.

$$V(X) = E(X^2) - \underline{E(X)^2}$$
$$m^2$$

Recurrence relation

Ex. Fibonacci:

$f_n = f_{n-1} + f_{n-2}$ $\quad f_0 = 0,\ f_1 = 1$

$f_n = g(n)$ $\quad f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

$n=0$ $\quad f_0 = 0$

$n=1$ $\quad f_1 = 1$

$n=2$ $\quad f_2 = \dots = 1$

$n=3$ $\quad f_3 = \dots = 2$

⋮