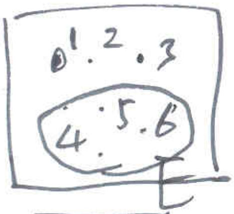
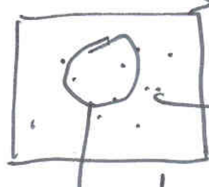
4/27 Discrete Probability

Sample space S

Ex) 주사위 ~~die~~





outcomes s

Event. E
(사건)

$s \in S$

$E \subseteq S$

$s \in E$.

Def. $\boxed{\text{finite}}$ $p(E) = \frac{|E|}{|S|}$ infinite $|\mathbb{N}| = |E|$

Ex2. $|S| = 36$ $(i, j) \in S$ $1 \le i, j \le 6$

$\{E\}_{sum=7} = \{(1,6), (2,5), \ldots, (6,1)\}$

$|E| = 6$

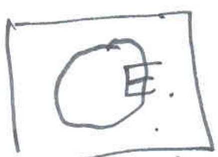
$p(E) = \frac{6}{36} = \frac{1}{6}$

Poker

Combination of Events

~~Def~~ . Complementary of Event E

$\bar{E}$



union of Event E_1 and E_2

$p(E_1 \cup E_2)$

$= p(E_1) + p(E_2) - p(E_1 \cap E_2)$

Ex)7. $P(E_2 \cup E_5) = P(E_2) + p(E_5) - P(\underbrace{E_2 \cap E_5}_{E_{10}})$

$= 0.50 + 0.20 - 0.10$

$= 0.6$.

infinite. (X)

not equal prob.

3/ 6.2 Probability Theory

S: countable sample space

$s \in S$: outcome. Real

$\cancel{p(s)}$ $p: S \to [0, 1]$ prob. dist.

$$\boxed{\begin{array}{ll} \text{i) } \forall s \in S, 0 \le p(s) \le 1 & p(s) \in \mathbb{R}. \\ \text{ii) } \sum_{s \in S} p(s) = 1. & \end{array}}$$

Ex 1. $p(H) = p(T) = \frac{1}{2}$ $\{H, T\}$ $(\frac{1}{2} + \frac{1}{2} = 1)$

but $p(H) = 2p(T)$ $p(H) = \frac{2}{3}$, $p(T) = \frac{1}{3}$ $(\frac{2}{3} + \frac{1}{3} = 1)$

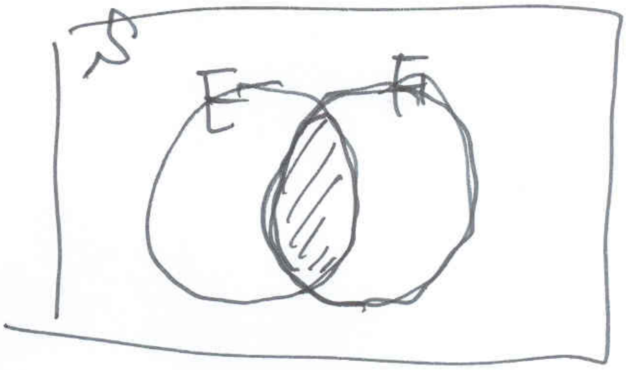
$$p(E) = \sum_{s \in E} p(s)$$

$$p(\bar{E}) = 1 - p(E)$$

$$= \sum_{s \in \bar{E}} p(s) = \sum_{s \notin E} p(s) = 1 - \sum_{s \in E} p(s) = 1 - p(E).$$

$p(E_1 \cup E_2) = \cdots$

Conditional probability



$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

$$p(F|E) = \frac{p(E \cap F)}{p(E)}$$

Ex 2 $S = \{0000, \ldots, 1111\}$

E at least 2 consecutive 0's.

F first bit is 0.

$E \cap F = \{0000, 0001, 0010, 0011, 0100\}$ 5개

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{\frac{5}{16}}{\frac{1}{2}} = \frac{5}{8}$$

3) Bayes Theorem

$$P(F|E) = \frac{P(F\cap E)}{P(E)} \qquad P(E/F) = \frac{P(E\cap F)}{P(F)}$$

$$\therefore P(F|E)\cdot P(E) = P(E|F)\cdot P(F) = P(E\cap F)$$

$$P(F|E) = \frac{P(E|F)\cdot P(F)}{P(E)}$$

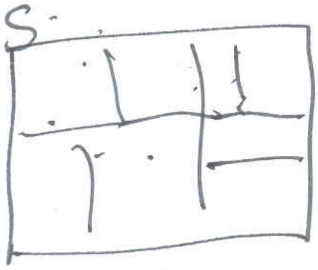
$$P(F|E) = \frac{P(E\cap F)}{P(E)} \qquad P(E\cap F) = P(E)\cdot P(F)$$

$$P(F|E) = \frac{\cancel{P(E)}\cdot P(F)}{\cancel{P(E)}} = P(F)$$

Def 4 E and F are independent if

$$P(E\cap F) = P(E)\cdot P(F)$$

S



partition

$X : S \to A$ (?)

$s \in S$

vs $2^S \to A$

$E \in 2^S$

$\subseteq S$

Distribution of random variable X

pair $(s, p(X=s))$

$p(X=s)$

$$E(X) = \sum_{s\in S} p(s)\cdot X(s) \in A$$

?

주사위

$$\frac{1}{6}\cdot 1 + \frac{1}{6} 2 + \cdots + \frac{1}{6}\cdot 6$$

$$= \frac{21}{6} = \frac{7}{2} = 3.5$$

24

윷놀이

도	$\frac{4}{16}$
개	$\frac{6}{16}$
걸	$\frac{4}{16}$
윷	$\frac{1}{16}$
모	$\frac{1}{16}$

4 12 12 4 5

$$= \frac{4\cdot 1 + 6\cdot 2 + 4\cdot 3 + 4 + 5}{16}$$

$$= \frac{37}{16} = 2.3\ldots$$

$$\begin{array}{r} 2.3 \\ 16\overline{)37} \\ 32 \\ \hline 50 \\ 48 \\ \hline 2 \end{array}$$