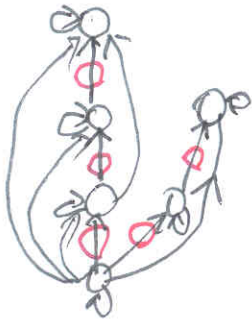


4/9 Poset & Program Verification.

Poset (A, R)

- $R \subseteq A \times A$
- ① (ir)reflexive
 - ② antisymmetric
 - ③ transitive

equiv. rel.
partition



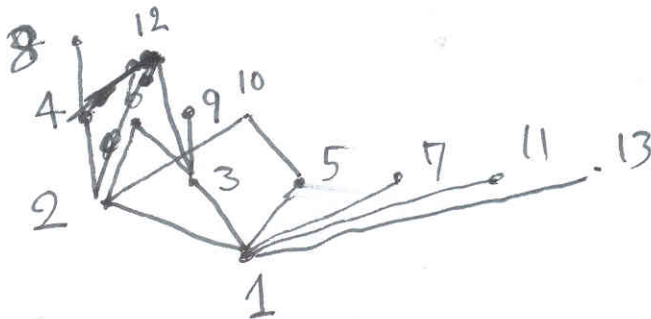
Hasse diagram

a cover b

$$a < b \wedge \nexists c \in A . a < c < b$$

$$a \leq c \wedge c \leq b$$

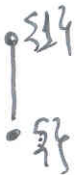
$(\mathbb{N}, |)$



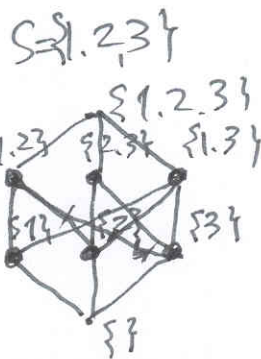
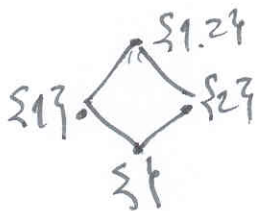
$$12 = 2^2 \cdot 3^2$$

$$24 = 2^3 \cdot 3$$

$(2^S, \subseteq)$
 $S = \{1\}$



$S = \{1, 2\}$



$S = \{1, 2, 3, 4\}$ $S = \{1, 2, 5\}$

(\mathbb{N}, \leq)



Lattice : unique lub, glb.

$$(A, \leq) \text{ lattice} \longrightarrow (A, \wedge, \vee)$$

$$\text{lub} : A \times A \rightarrow A$$

$$\wedge, \vee : A \times A \rightarrow A$$

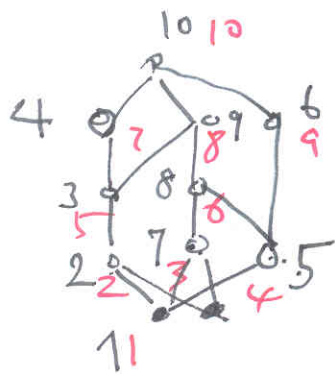
$$\text{glb} : A \times A \rightarrow A$$

binary operator on A .

$$\text{lub } \{2, 3, 5\} \{3, 6, 7\} \longrightarrow \{2, 3, 5, 6, 7\}$$

$$\{3\}$$

poset (A, \leq) ... numbering
 . total order, successor



how vs what

Tarjan's Depth first Search algorithm

