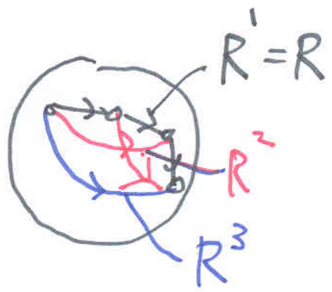
1) 4/4 Closure of relations & Equivalence relation, poset. ①

ref. closure of R

$R \cup id_A$

symmetric closure of R

$R \cup R^{-1}$



transitive closure of R ?

Thm 1. let $R \subseteq A \times A$. R is transitive $\leftrightarrow$ $R^n \subseteq R$ $\forall n \in \mathbb{N}^+$.

i) $R^n \subseteq R$ $\forall n \in \mathbb{N}^+$ $\rightarrow$ R is transitive.

$\forall (a,b) \in R, \forall (b,c) \in R \rightarrow (a,c) \in R$

$\searrow (a,c) \in R^2$, $R^2 \subseteq R$

$(a,d) \in R^3, R^5, R^9, \ldots$

ii) R is transitive $\rightarrow$ $R^n \subseteq R$, $\forall n \in \mathbb{N}^+$.

$(P \rightarrow Q :\ Q = T :$ trivial$)$

$\quad\quad\quad\ P = F :$ vacous$)$

basis) $n = 1$. $R^1 \subseteq R$

ind.) Assum $R^n \subseteq R$, R is tran.

$\boxed{\forall (a.b) \in R^{n+1}}$, $\exists c \in A$ $\exists (a.c) \in R$, $(c.b) \in R^n$,

Since $R^n \subseteq R$, $(c.b) \in R$

$\boxed{(a,b) \in R}$, $\boxed{R^{n+1} \subseteq R}$

($\because$ transitive)

$\boxed{\forall a \in A \rightarrow a \in B \leftrightarrow A \subseteq B}$

- Relational DB
- Hierarchical DB
  - structure C or C++

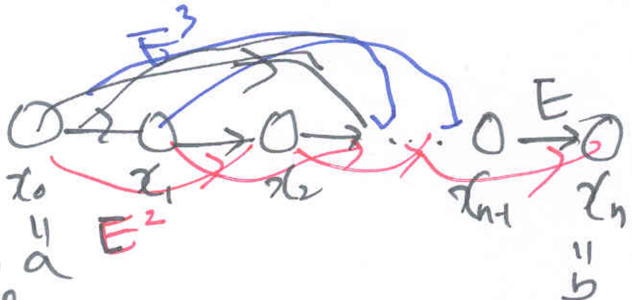
transitive closure of R

$$R^+ = R^1 \cup R^2 \cup R^3 \cup \cdots$$
$$= \bigcup_{i \in \mathbb{N}^+} R^i$$

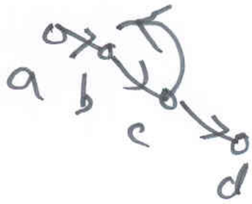
2) Path of length n in directed graph $G=(V,E)$ 12

where $E \subseteq V \times V$

(E: binary relation on V.



cycle.



$(a,b,c,d,)$

~~$(a,b)(b,c),(c,d)$~~

$a \le b \le c \le d$

$(a \le b \wedge b \le c \wedge c \le d)$

a b c b c d

⋮

Thm 1. path of length n from a to b

$\longleftrightarrow (a,b) \in R^n$

$R^+ = \{(a,b) \mid (a,b) \in R^n, \forall n \ge 1\}$

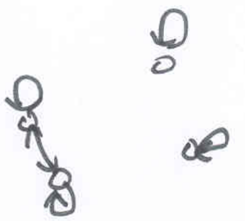
$= R^1 \cup R^2 \cup R^3 \cdots$

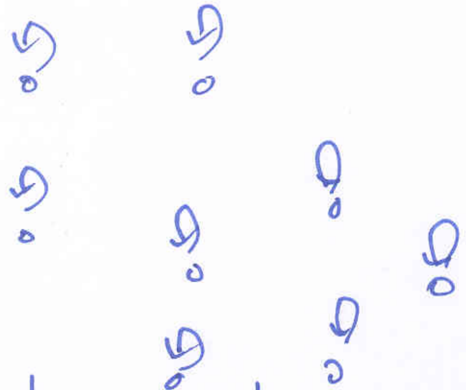
$= \bigcup_{i=1}^{\infty} R^i = \bigcup_{i \in \mathbb{N}^+} R^i$

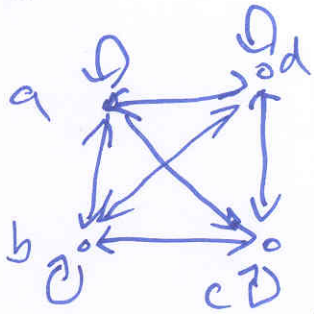
$R^* = R^+ \cup id_A$.

$= R^0 \cup R^1 \cup R^2 \cup \cdots$

$= \bigcup_{i=0}^{\infty} R^i = \bigcup_{i \in \mathbb{N}} R^i$







	a	b	c	d
a	1	1	1	1
b	1	1	1	1
c	1	1	1	1
d	1	1	1	1

Let A be a set

$\text{Partion}(A) = \{A_i \mid i \in I\}$

$\forall i \in I, A_i \ne \emptyset$ — nonempty

$\bigcup_{i \in I} A_i = A$ — exhaustive

$A_i \cap A_j = \emptyset$ — disjoint

$i,j \in I, i \ne j$