4/2. Chap 8. Relations.

8.1 Relations and their properties

Def. 1 Let A and B be two sets.

A binary relation R from A to B

$$R \subseteq A \times B$$

A ↑ domain, B ↑ range / codomain.

$\underline{(a,b) \in R}$ or aRb

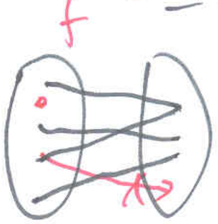
$(a,b) \notin R$, $a \not R b$

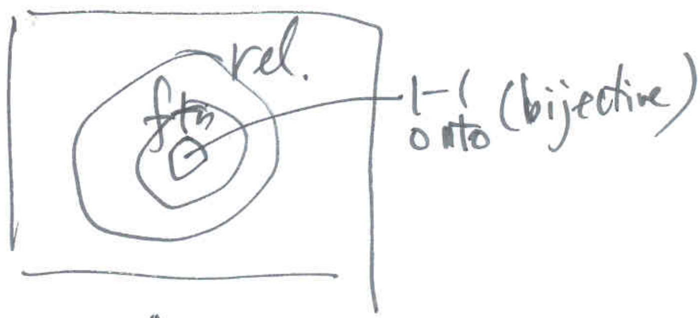
$\boxed{A \times B = \{(a,b) \mid a \in A,\ b \in B\}}$

Ex. $\leq \subseteq \mathbb{R} \times \mathbb{R}$. $3 \leq 7$, $4 \not\leq 3$

$3 \leq \pi \ldots$

Function
[total
[uniqueness





$R \subseteq A \times A$ relation on A.

reflexive $3 \leq 3$

irreflexive $3 \not< 3$

$5 = 5$

$5 \neq 4$

$\leq \subseteq \mathbb{R} \times \mathbb{R}$

$= \subseteq \mathbb{R} \times \mathbb{R}$

$< \subseteq \mathbb{R} \times \mathbb{R}$

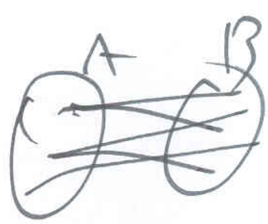
$3 < 5$

$5 \not< 3$

$\leq$ – transitive?

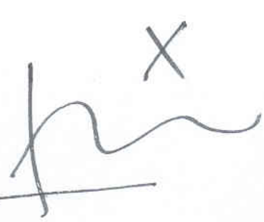
$=$ – ?

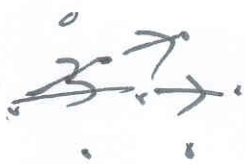
$\leq \;=\; < \cup =$



Relation $\underline{\text{directed}}$

① graph





② boolean matrix.

$R \subseteq A \times B$

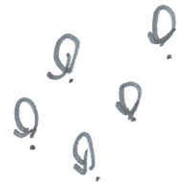
$|A| = n \quad |B| = m$

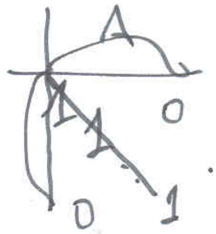
m_{ij} [$1 \leq i \leq n$, $1 \leq j \leq m$

Def $\left[\begin{array}{lll} R^1 = R & \text{basis} & n=1 \\ R^{n+1} = R^n \circ R & \text{recursion} & n \geq 2 \end{array}\right] \; n \geq 1 \quad n \in \mathbb{N}^+$

$$R^3 =_r R^2 \cdot R =_r R^1 \cdot R \cdot R =_b R \cdot R R$$

Def $id_A = (\Delta =) \{(a,a) \mid a \in A\} \subseteq A \times A$

$I_A =$ 



Def $\left[\begin{array}{ll} R^0 = id_A & n=0 \\ R^{n+1} = R^n \circ R & n \geq 1 \end{array}\right] > n \geq 0 \quad n \in \mathbb{N}_0$

$$R^3 =_r R^2 \cdot R =_r R^1 \cdot R \cdot R = R^0 \cdot R \cdot R \cdot R = \underbrace{id_A} \cdot R \cdot R \cdot R$$
$$= R \cdot R \cdot R$$

Def. $R \subseteq A \times A$. R may or may not be $\mathbb{P}$ = {reflexive, symmetric, transitive}.

$\forall T \subseteq A \times A$ with ~~property~~ $\mathbb{P} \in \mathbb{P}$ $R \subseteq T$, smallest T.

reflexive closure of $R = R \cup id_A$

symmetric " of $R = R \cup R^{-1}$

transitive " of $R^{\dagger} =? R^1 \cup R^2 \cup R^3 \cup \ldots$

dagga $= \bigcup_{i \in \mathbb{N}^+} R^i$

reflexive and transitive " of $R^* = \overset{id_A}{R^0} \cup R^1 \cup R^2 \cup \ldots$

star $= \bigcup_{i \in \mathbb{N}} R^i$