X 1

3/30. Recursion

Def. 2. Let Σ be a set of symbols.
(vocabulary, alphabet)

seq. →	
symbol $a \in \Sigma$	string $x \in \Sigma^*$
vocabulary Σ	lang. $L \subseteq \Sigma^*$

set

Basis $\varepsilon \in \Sigma^*$. $(|\varepsilon| = 0)$

Rec. $x \in \Sigma^*, a \in \Sigma \rightarrow xa \in \Sigma^*$ $(|xa| \geq 1)$

Another def for Σ^*.

$$\bigcup_{i \in \mathbb{N}_0} \Sigma^i \quad \text{where} \quad \begin{cases} \Sigma^0 = \{\varepsilon\} \\ \Sigma^i = \Sigma^{i-1} \cdot \Sigma \end{cases}$$

$\forall x \in \Sigma^*$
$\varepsilon \cdot x = x \cdot \varepsilon = x$
$\forall L \subseteq \Sigma^*$
$\{\varepsilon\} \cdot L = L \cdot \{\varepsilon\} = L$
but $\phi L = L \cdot \phi = \phi$

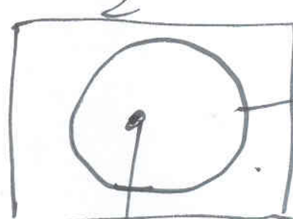
$*$ ε - epsilon (e)
λ : lambda (l)

length of string
$\Sigma^* \rightarrow \mathbb{N}$
basis: $|\varepsilon| = 0$
ind. $x \in \Sigma^*, a \in \Sigma$. then $|xa| = |x| + 1$

Natural number

$$\begin{cases} 0 \in \mathbb{N} \\ n \in \mathbb{N} \rightarrow n+1 \in \mathbb{N} \\ \text{no more} \end{cases}$$

Σ^*



$L \subseteq \Sigma^*$ language

$x \in \Sigma^*$ string

if $x \in L$ then x is called sentence of L.

$x \overset{?}{\in} L$ membership problem

Well-formed formulae
↳ syntax (文法)

$\Sigma = \{\cancel{T}, \cancel{F}, \neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$
$\cup V = \{a, b, \ldots, z\}$

$TT \in \Sigma^*$ $\quad p \rightarrow q \in \Sigma^*$
$\rightarrow a \in \Sigma^*$

2)

$$E \rightarrow \underbrace{T \mid F \mid S}_{\text{basis}} \mid \neg E \mid E \wedge E \mid E \vee E \mid E \rightarrow E \mid E \leftrightarrow E \mid (E)$$

prop. variables

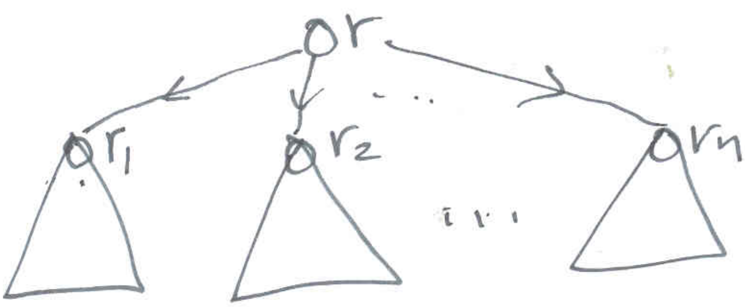
$\neg T, \neg F, \neg S, \neg\neg T, \neg\neg\neg T, \ldots$

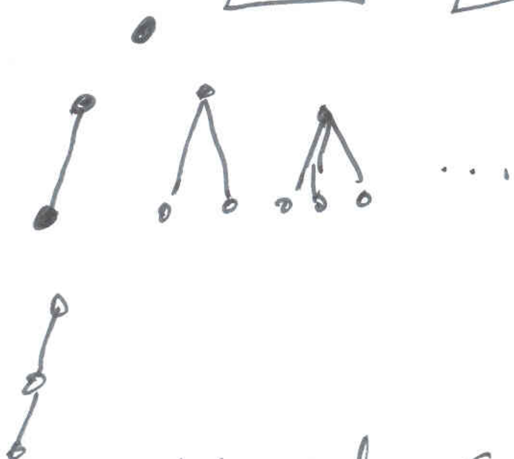
$p \wedge q$

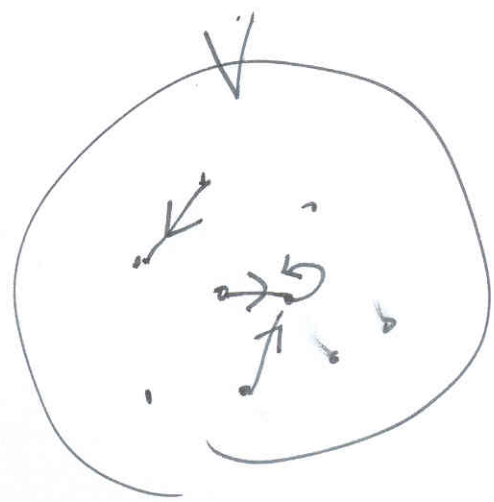
Well-formed formulae

Def. 4 The set of rooted tree. (tree)

basis







A directed graph $G = (V, E)$

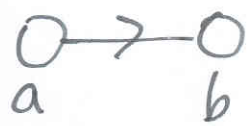
V : set of vertices

ex

E : set of edges $(a, b) \in E$

$E \subseteq V \times V$ ↓ $a, b \in V$

binary relation on V



$T(V, E, r)$

i) V, ii) $V \times V \subseteq E$ iii) $r \in V$.

3) Rooted-tree

basis $(\{r\}, \{\}, r)$ is a rooted tree.

recur. Let $T_1 = (V_1, E_1, r_1), T_2 = (V_2, E_2, r_2), \cdots, T_n = (V_n, E_n, r_n)$ are rooted trees. (disjoint)

$$\left(\bigcup_{i=1}^{n} V_i \cup \{r\}, \bigcup_{i=1}^{n} E_i \cup \{(r, r_1), (r, r_2), \cdots (r, r_n)\}, r \right)$$

where $r \notin \bigcup_{i=1}^{n} V_i$ new