1) 3/ Recursion vs Induction 1

Generalized induction

$\exists C \geq 1$ $\quad P(c) \wedge P(c+1) \wedge \cdots \wedge P(c+C-1)$ ..... C-basis

$\forall n: n \geq c,\ P(n) \rightarrow P(n+C)$

$\therefore \forall n \geq c, P(n)$. $\quad P(c+C) \wedge P(c+C+1) \wedge \cdots\ P(c+2C-1)$

$P(c+2C)$

우표(stamp) 5¢, 4¢,

12¢ ≤ n $\quad P(n)$

$P(12)$
$P(13)$
$P(14)$
$P(15)$

Basis

$P(16)$

$P(19)$
$P(20)$

$P(n) \rightarrow P(n+1)$ (X)

$P(n) \rightarrow P(n+4)$

induction

4.3 Recursive Definition & structural Induction

Recursion.
Induction

Recursively defined function

$f: \mathbb{N} \rightarrow S$ (any set S) ⟶ sequence

$\mathbb{N}$ (sequence of numbers)

V (Vocabulary : Seq. of symbols)
string

점화식

i) Define $f(0)$ or $f(1)$ or $f(c)$.. initial point. $\quad f(1)$

ii) Define $f(n)$ in terms of ($\cancel{f(0), f(1), \ldots}$, $f(n-1)$ $f(n-2)$)

recurrence relation

2)

$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n\ \text{times}}$

iterative def.

$a^1 = a$

$a^2 = a \cdot a$

$a^3 = a \cdot a \cdot a$

$\vdots$

$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n}$

recursive def.

$$\begin{cases} a^1 = a & (n=1) \quad \text{basis} \\ a^n = a^{n-1} \cdot a & (n \geq 2) \quad \text{recursion} \end{cases}$$

$a^4 =_r a^3 \cdot a =_r a^2 \cdot a \cdot a =_r a^1 \cdot a \cdot a \cdot a$

$=_b a \cdot a \cdot a \cdot a$.

$$\begin{cases} a^0 = 1 & (n=0) \\ a^n = a \cdot a^{n-1} & (n \geq 1) \end{cases}$$

$a^2 =_r a \cdot a^1 =_r a \cdot a \cdot a^0 = a \cdot a \cdot 1$

$= a \cdot a$

Def fibonacci series.

$f_0 = 0,\ f_1 = 1,\ f_2 = 1,\ f_3 = 2,\ f_4 = 3,\ f_5 = 5,\ f_6 = 8, \ldots$

$f_n = f_{n-1} + f_{n-2}$

$f_n < 2^n$

Recursively Defined sets and structures

Σ^* : Σ: alphabet : set of symbols.

↳ set of strings over Σ.

$\Sigma = \{0, 1\}$

i) $\varepsilon \in \Sigma^*$ —— basis

ii) $\forall w \in \Sigma^* \wedge \forall a \in \Sigma \longrightarrow wa \in \Sigma^*$. —— recur.

($\because$ $w \cdot \varepsilon = \varepsilon \cdot w = w$)

$\forall w \in \Sigma^*$

iii) No more

~~개념이~~ . ~~[illegible]~~

$\Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, \ldots$

ε : empty string

identity for concatenation

w	X	ε	ε	0	0	1	1	00	00	01	01				
a	X	0	1	0	1	0	1	0	1	0	1				
wa		0	1	00	01	10	11	000	001	010	011	100	101	110	111 , ...

3/ ~~Another def. for~~ Σ^* over Σ

$$\Sigma^* = \bigcup_{i \in \mathbb{N}_0} \Sigma^i = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \cdots$$

where Σ^i ~~is~~ defined recursively as follows

$$\begin{cases} \Sigma^0 = \{\varepsilon\} \\ \Sigma^i = \Sigma \cdot \Sigma^{i-1} \end{cases}$$ where $A \cdot B = \{xy \mid x \in A, y \in B\}$

$\rightarrow \Sigma^i = \underbrace{\Sigma \cdot \cdots \Sigma}_{i \text{ 개}}$

Ex) $\Sigma = \{0, 1\}$

$\Sigma^0 = \{\varepsilon\}$ $\Sigma^1 = \Sigma \cdot \Sigma^0 = \Sigma \cdot \{\varepsilon\} = \Sigma = \{0, 1\}$, $\Sigma^2 = \{00, 01, 10, 11\}$, $\Sigma^3 =$

Empty string

identity for concatenation

$\varepsilon \neq \emptyset$ $\{\varepsilon\} \neq \emptyset$

setx set   set set

$w \in \Sigma^*$

$\varepsilon \cdot w = w \cdot \varepsilon = w$

$L \subseteq \Sigma^*$

$\{\varepsilon\} \cdot L = L \cdot \{\varepsilon\} = L$

$\emptyset \cdot L = L \cdot \emptyset = \emptyset$