3/26 Prime, gcd, lcm, & Chap 4 Recursion

o o o o o
2 3 5 7 11 ....

The First Principle of Mathematical Induction

$$\frac{P(0) \qquad \forall n\geq 0,\ P(n) \rightarrow P(n+1)}{\forall n \geq 0,\ P(n)}$$

$\underline{P(0)}$

$\underline{P(0)} \rightarrow \underline{P(1)}$

$P(1) \rightarrow P(2)$

$\vdots$

$P(n): \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$$\frac{n \geq 0 \qquad \frac{0(0+1)}{2} = 0}{n \geq 1 \qquad \frac{1(1+1)}{2} = 1}$$

if we define $\boxed{\sum_{k=1}^{0} k \triangleq 0}$! X

$\sum_{k=1}^{1} k = 1$

basis

i) $\frac{n=0}{n=1}$:

ii) $\frac{\forall n \geq 0}{\forall n \geq 1}$ $P(n) \rightarrow P(n+1)$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n+1} k = \frac{n(n+1)}{2} + n+1 = \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2}$$

$\therefore$ Q.E.D.

The Second Prin. of M.I.

$P(0)$

$P(0) \rightarrow P(1)$

$P(0) \wedge P(1) \rightarrow P(2)$ vs $P(n) \rightarrow P(n+1)$

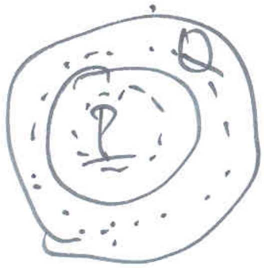
$P(0) \wedge P(1) \wedge P(2) \rightarrow P(3)$

$\vdots$

$P(0) \wedge \cdots \wedge P(n) \rightarrow P(n+1)$

$p \to q$ vs $p' \to q$ when $(p' \Rightarrow p.)$

truth set



Prove $P \subseteq Q$

$\therefore p \to q$

$p(n) \to p(n+1)$ (X)

$p(n) \to p(n+\cancel{2}\,c)$ (O)

p287 Ex. 4

4¢, 5¢-stamp 4, 5, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19

4¢ 4¢

	12	13	14	15
4¢	3	2	1	0
5¢	0	1	2	3

Basis (4)

P(12).

P(13).

P(14).

P(15).

Induction $\bigwedge_{\forall n \geq 12} P(n) \to P(n+4)$