Halting Problem & Big O notation

Halting problem

terminate or not?

halting problem

$f: \mathbb{N} \to \{T, F\}$

function from $\mathbb{N}$ to boolean: Problem

$\updownarrow$ 1:1

infinite binary string

$\updownarrow$ 1:1

Power sets of Natural numbers

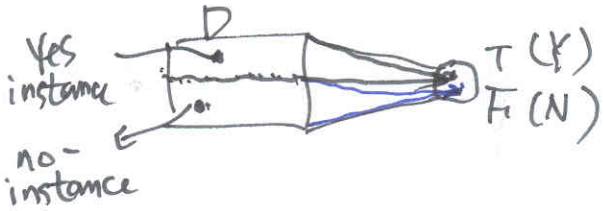
Problem $\updownarrow$ membership problem

prob.

$|2^{\mathbb{N}}|$ ... uncountable

alg. or program

$|V^*|$ ... countable



$$V^* = \bigcup_{i \in \mathbb{N}_0} V^i \qquad V = \{0, 1\}$$

$$= V^0 \cup V^1 \cup V^2 \cup V^3 \cup \ldots$$

$\{\varepsilon\}$ $\{0,1\}$ $\{00, 01, 10, 11\}$ $\to \{000, \ldots, 111\}$

$V^* \longleftrightarrow \mathbb{N}$

1:1 onto

- Cantor's diagonal arg.
- Russel's paradox
- Halting problem

3.2 The groth of function

$f: \mathbb{N} \to \mathbb{R}$

$\mathbb{N}$ $\mathbb{R}$

$f(x)$ is $O(g(x))$ if $\exists C, k \in \mathbb{R}^+$ s.t. $|f(x)| \leq C \cdot |g(x)|$

Ex 1. $f(x) = 100x^2 + 2x + 1$ is $O(x^2)$ $\forall x > k$

$C = 4, k = 1$

400

is $O(x)$ X

$O(x^3)$ O